1 Introduction

Whereas typical transmissions that utilize combinations of gears can only adopt a fixed number of discrete transmission ratios, a continuously variable transmission, or simply CVT, can adopt any arbitrary gear ratio. Whereas typical transmissions utilize toothed gears, the CVT employs a sphere in rolling contact with a set of rollers; loads applied to the CVT are supported across these rolling contacts, resulting in microslips of varying amounts at each contact area. In this paper, we describe the causes of microslips in the CVT and ways to lessen them through an alternative CVT design. [DOI: 10.1115/1.2803711]

The continuously variable transmission (CVT) is a type of transmission that can adopt any arbitrary gear ratio. Whereas typical transmissions utilize toothed gears, the CVT employs a sphere in rolling contact with a set of rollers; loads applied to the CVT are supported across these rolling contacts, resulting in microslips of varying amounts at each contact area. In this paper, we describe the causes of microslips in the CVT and ways to lessen them through an alternative CVT design.

The design of the CVT utilizes a sphere that is surrounded and held in place by four rollers. Two of the four rollers, called steering rollers, constrain the sphere to rotate about an axis of rotation, whose direction can be changed by adjusting the orientations of the steering rollers. The relative velocities of the remaining two rollers, called drive rollers, are dependent on the orientation of the sphere’s axis of rotation.

1.2 Motivation. The motivation for our analysis of the CVT stemmed from our physical interaction with existing cobots. We observed from our analysis of the cobots that the measured velocity ratios of a pair of joints, connected by CVTs, differed noticeably from the intended velocity ratios.

Ideally, the ratio of the angular velocities of a pair of robotic joints, connected by CVTs, is the same as the transmission ratios of the CVTs. In practice, however, the actual velocity ratio often deviates from the intended ratio. One contributor to this is microslip (or creep) across the rolling contacts between the CVT sphere and the rollers; a force on a robot’s end point produces reactive forces across the rolling contacts, thereby giving rise to microslip (or creep) at these rolling contacts.

Herein, we develop a model of the CVT microslip, verify the model with experiments, and then use the model to simulate a proposed improvement to the CVT design.

1.3 Related Work. Akehurst et al. [4] give a good overview of published works related to CVTs that operate through rolling traction. Previous works concerning microslips in CVTs, utilized in cobots, are limited. They include the kinematic creep model [5] of Gillespie et al. and the experimental analysis [6] of Brokowski et al. of the CVT.

In Ref. [5], Gillespie et al. model the drive rollers as rigid bodies that make line contacts with the sphere. They describe that the drive rollers transmit longitudinal forces across the rolling contacts while under a state of spin. In their work, Gillespie et al. determine an expression for the velocity ratio as a function of the transmission angle, load, and spin. The work of Gillespie et al. is limited to the analysis of slips at contacts between the sphere and the steering rollers; their work does not examine the causes and effects of slips at the contacts between the sphere and the steering rollers.


Spin refers to relative angular motion of the normal of two bodies in contact.
rollers.

The work of Brokowski et al. involves subjecting a physical CVT to experimental testing and comparing their results to the kinematic creep model of Gillespie et al. In their work, Brokowski et al. thoroughly examine the mechanics of the contacts between the sphere and the drive rollers, but like Gillespie et al., Browkowski et al. also neglect to examine causes and effects of slips of the contacts between the sphere and the steering rollers.

1.4 Preview of Sections. Our work concerns an experimental as well as an analytical analysis of the CVT. Through experimental analysis, we develop a kinetic model that describes the CVT’s ability to produce the desired velocity ratios in the face of loads across the device. In addition, we develop an analytical model that describes the causes of microslips in the CVT and we offer suggestions for the design of an improved CVT.

In Sec. 2, we describe the design and kinematics of the CVT. In Sec. 3, we present an analytical model that describes the microslips in the CVT. In Sec. 4, we verify our model through experiments on a CVT that is subject to various loads. Finally, in Sec. 5, we describe the suggested designs of an improved CVT.

2 Continuously Variable Transmission

2.1 Continuously Variable Transmission Kinematics. The design of the CVT utilizes four rollers in rolling contact with a sphere (Fig. 2). Two of these rollers, called the steering rollers, constrain the sphere to rotate about a particular axis of rotation. The relative motions between the remaining two rollers, called the drive rollers, are dependent on the orientation of the sphere’s axis of rotation.

In Fig. 3, the three-dimensional coordinate frame \( N \), with coordinate axes \( x, y, \) and \( z \), is fixed to the base of the CVT. The unit vectors \( d_1, d_2, \) and \( d_3 \) are established by rotating the coordinate frame \( N \) about the \( z \)-coordinate axis by 45 deg. The three orthogonal unit vectors \( e_i (i=1,2,3) \) are established by rotating the frame \( N \) about the \( x \)-coordinate axis by 135 deg. The unit vector \( e_3 \) is in the positive \( x \)-coordinate direction. The unit vectors \( e_1 \) and \( e_2 \) are defined using the right-hand rule. The center of the sphere \( O \) is located at the origin of the coordinate frame \( N \).

Figure 3(a) shows the drive Rollers \( D_1 \) and \( D_2 \). The center of the contact patch between the sphere \( O \) and the Roller \( D_1 \) lies on an axis that is collinear to the unit vector \( d_1 \). Drive Roller \( D_1 \) has an angular velocity \( \omega_1 \) about an axis that is parallel to the unit vector \( d_1 \). The center of the contact patch between \( O \) and \( D_2 \) lies on an axis that is collinear to the unit vector \( d_2 \). Drive Roller \( D_2 \) has an angular velocity \( \omega_2 \) about an axis that is parallel to the vector \( d_2 \).

Figure 3(b) shows the steering Rollers \( S_1 \) and \( S_2 \) and the steering Forks \( C_1 \) and \( C_2 \). \( C_1 \) rotates about an axis that is collinear to the unit vector \( e_1 \). The center of the contact patch between the sphere \( O \) and the Roller \( S_1 \) lies on an axis that is collinear to the unit vector \( e_1 \). \( C_2 \) rotates about an axis that is collinear to the unit vector \( e_2 \). The center of the contact patch between \( O \) and the Roller \( S_2 \) lies on an axis that is collinear to the unit vector \( e_2 \). \( C_1 \) and \( C_2 \) are mechanically coupled via a bevel gear (not shown) such that the orientations of \( C_1 \) and \( C_2 \) can be described by the same steering angle \( \phi \). The coordinate axis \( s_1 \) is fixed to \( C_1 \) and the coordinate axis \( s_2 \) is fixed to \( C_2 \). Steering Roller \( S_1 \) has an angular velocity \( \omega_{s_1} \) about \( s_1 \) and steering Roller \( S_2 \) has an angular velocity \( \omega_{s_2} \) about \( s_2 \).

Figure 4(a) shows the sphere \( O \) and steering Roller \( S_1 \); steering Roller \( S_2 \) and the two drive rollers are not shown. Figure 4(b) shows the sphere and the steering Roller \( S_2 \); steering Roller \( S_1 \) and the two drive rollers are not shown. Let \( r \) be the radius of the steering rollers and let \( R \) be the radius of the sphere \( O \). Then, the rolling constraint between the Roller \( S_1 \) and \( O \), whose velocity is \( \Omega \), requires that

\[
\Omega \times R e_i = (\omega_1 \cos \phi e_2 + \omega_2 \sin \phi e_3) \times (-r e_i) \quad (1)
\]

Also, the rolling constraint between the Roller \( S_2 \) and \( O \) requires that

\[
\Omega \times R e_2 = (\omega_1 \cos \phi e_1 + \omega_2 \sin \phi e_3) \times (-r e_2) \quad (2)
\]

Combining Eqs. (1) and (2), we have
Or we may write

\[ \Omega = \frac{r_o \omega_o}{R} (\cos \phi \mathbf{e}_1 - \cos \phi \mathbf{e}_2 - \sin \phi \mathbf{e}_3) \]  

(3)

Or we may write

\[ \Omega = \frac{r_o \omega_o}{R} \left( \frac{\sqrt{2} \cos \phi - \sin \phi}{\sqrt{2}} \mathbf{d}_1 + \frac{\sqrt{2} \cos \phi + \sin \phi}{\sqrt{2}} \mathbf{d}_2 \right) \]  

(4)

Fig. 4 Two additional views of the CVT. The orientations of both steering rollers are described by steering angle \( \phi \).

We see that from the above equation, the sphere’s instantaneous axis of rotation lies on a plane defined by the unit vectors \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \).

Let \( \gamma \) be an axis that is collinear to the sphere’s instantaneous axis of rotation (Fig. 5) and let \( \gamma \) describe the angle between the unit vector \( \mathbf{d}_1 \) and the axis \( \gamma \). Then, \( \gamma \), called the CVT angle, is related to the steering angle \( \phi \) by

\[ \gamma = \tan^{-1} \left( \frac{\sqrt{2} + \tan \phi}{\sqrt{2} - \tan \phi} \right) \]  

(5)

Let \( \omega_o \) be the sphere’s angular velocity about the axis \( \gamma \). Then, the rolling constraint between the Roller 1 and the sphere \( O \) requires that

\[ -\omega_o \mathbf{d}_1 \times \mathbf{r} = (\omega_o \cos \gamma \mathbf{d}_1 + \omega_o \sin \gamma \mathbf{d}_2) \times \mathbf{r} \]  

(6)

Also, the rolling constraint between the Roller 2 and the sphere \( O \) requires that

\[ -\omega_o \mathbf{d}_2 \times \mathbf{r} = (\omega_o \cos \gamma \mathbf{d}_1 + \omega_o \sin \gamma \mathbf{d}_2) \times \mathbf{r} \]  

(7)

Equations (6) and (7) can be combined to find the relationship between the velocities of the drive rollers and the CVT angle \( \gamma \):

\[ \omega_1 = \omega_o \mathbf{v}_1 + \omega_o \mathbf{v}_2 \]  

(10)

Or it may be expressed in the coordinate frame \( U \):

\[ \omega = \omega_o \mathbf{u}_1 + 0 \mathbf{u}_2 \]  

(11)

where \( \omega_o \), called the CVT parallel velocity, describes the velocity of the CVT along its allowed direction of motion.

Given that frame \( U \) is rotated \( \gamma \) degrees from frame \( V \), the velocities of the drive rollers may be written as

\[ \omega_1 = \omega_o \cos \gamma \]  

(12)

\[ \omega_2 = \omega_o \sin \gamma \]  

(13)

Fig. 5 The sphere’s instantaneous axis of rotation is described by the CVT angle \( \gamma \).

2.1.1 Continuously Variable Transmission Velocity Vector. We may combine Eqs. (5) and (8) to find an alternative expression of the ideal transmission law:

\[ \frac{\omega_2}{\omega_1} = \frac{\sqrt{2} + \tan \phi}{\sqrt{2} - \tan \phi} \]  

(9)

Equation (8) is called the ideal transmission law. We may combine Eqs. (5) and (8) to find an alternative expression of the ideal transmission law:

\[ \frac{\omega_2}{\omega_1} = \frac{\sqrt{2} + \tan \phi}{\sqrt{2} - \tan \phi} \]  

(9)

Fig. 6 Two-dimensional Cartesian coordinate system describing the velocities of the drive rollers.

2.2 Continuously Variable Transmission Slip Angle. In practice, the ratio of the velocities between the two CVT drive rollers differs from that provided by the ideal transmission law (Eq. 8). We describe this difference between the ratio given by Eq. (8) and the measured, or the actual, ratio of the velocities by an angle in the coordinate frame \( V \).

We augment the two-dimensional Cartesian coordinate frame \( V \) with alpha (Fig. 7), called the CVT slip angle. Again, let the vector \( \omega \) describe the velocities of both drive rollers. In practice, \( \alpha \) deviates from the longitudinal velocity vector \( \mathbf{u}_1 \) such that it has
components along both the allowed and disallowed (\( u_\perp \)) directions of motion. Then, the velocity of the CVT is written as

\[
\omega = \omega_u u_1 + \omega_\perp u_\perp
\]

(14)

where \( \omega_\perp \) is the vector component of the CVT’s velocity vector along the disallowed direction of motion.

Let the angle \( \gamma \) describe the direction of the measured, or the actual, \( \omega \). Then, the difference between angles \( \gamma_m \) and \( \gamma \) yields the CVT slip angle (\( \alpha \)):

\[
\alpha = \gamma_m - \gamma
\]

(15)

2.3 Torque Balance. Let \( \tau_1 \) and \( \tau_2 \) be the torques by the drive Rollers \( D1 \) and \( D2 \), respectively. Then, from Eq. (8), the torque balance equation for the CVT is

\[
\frac{\tau_1}{\tau_2} = -\tan \gamma
\]

(16)

2.3.1 Lateral and Parallel Torques. In practice, the pair of torques \( \tau_1 \) and \( \tau_2 \) often deviate from Eq. (16).

In Fig. 8, we express the pair of drive roller torques \( \tau_1 \) and \( \tau_2 \) as a single vector \( \tau = \tau_1 v_1 + \tau_2 v_2 \) in the coordinate frame \( V \), similar to that for \( \omega \) in Fig. 7.

Again, we augment coordinate frame \( V \) with coordinate frame \( U \) that is rotated about the origin by \( \gamma \) degrees. We can now express \( \tau \) as a sum of the vector components along \( u_1 \) and \( u_\perp \):

\[
\tau = \tau_1 u_1 + \tau_\perp u_\perp
\]

(17)

The scalar component along \( u_1 \), \( \tau_1 \), is called the parallel torque and the scalar component along \( u_\perp \), \( \tau_\perp \), is called the lateral torque. Parallel torque \( \tau_1 \) is the component of \( \tau \) that is responsible for overcoming the CVT’s internal friction and inertia and is also responsible for producing forward motion. Lateral torque \( \tau_\perp \) is the component of \( \tau \) that is supported internally by the CVT. \( \tau_\perp \) is also known as CVT load; it is the load that is applied across the CVT.

3 Modeled Slip Angles

Before we describe our CVT slip model, let us first describe two terms that we will use to develop our model.

3.1 Definitions. We will use the following two examples to describe longitudinal slip ratio and lateral slip angle.

3.1.1 Longitudinal Slip Ratio. A free-rolling wheel of radius \( r \), whose translational velocity is \( v \), has an angular velocity \( \omega = v/r \). When the same wheel of radius \( r \), whose translational velocity is again \( v \), is called upon to transmit a tractive force against another body, the wheel incurs microslip at the interface between the two bodies in contact such that the angular velocity of the wheel differs from \( \omega \) by \( \Delta \omega \). In this example, the longitudinal slip ratio (\( \psi \)) is the ratio between \( \omega \) by \( \Delta \omega \):

\[
\psi = \frac{\Delta \omega}{\omega}
\]

(18)

3.1.2 Lateral Slip Angle. Figure 9 illustrates a plan view of wheel \( W \) in rolling contact with ground. The \( x-y \) coordinate frame is attached to ground. In the absence of an external forces, \( W \) rolls in the direction along the vector \( u_c \), described by the orientation angle \( \gamma \) of \( W \). When the same wheel is subjected to an external force perpendicular to \( u_c \), the wheel rolls in the direction of a vector described by \( \gamma \) plus lateral slip angle \( \alpha_\perp \).

3.2 Modeled Steering Roller Slip Angle. The steering rollers constrain the CVT sphere to rotate about a particular axis of rotation.

A load across the CVT (\( \tau\_l \)) is supported internally within the CVT across four contacts between the sphere and the steering and drive rollers. The two drive rollers support tractive forces in the direction of rolling, and the two steering rollers support forces lateral to the direction of rolling (Fig. 10).

Let \( f_{11} \) be the lateral force imparted on the Roller \( S1 \) by the sphere \( O \). Then, from Ref. [7], we know that \( f_{11} \) is a function of \( \tau_1 \) and \( \phi \):

\[
f_{11} = \frac{\tau_1 \cos \gamma}{r^2 \cos \phi - \sin \phi}
\]

(19)

where \( r \) is the radius of the steering rollers and \( \gamma \) is the transmission angle. To simplify the above algebraic expression, both \( \gamma \) and \( \phi \) are used. Recall that \( \gamma \) and \( \phi \) are related as described in Eq. (5).

Let \( f_{21} \) be the lateral force imparted on the Roller \( S2 \) by the sphere \( O \). Then,

\[
f_{21} = \frac{\tau_1 \cos \gamma}{r^2 \cos \phi - \sin \phi}
\]

(20)

\( \gamma \)
Let $\alpha_{l1}^S$ be the lateral slip angle by the Roller S1 and let $\alpha_{l2}^S$ be the lateral slip angle by the Roller S2. Let us model the angles $\alpha_{l1}^S$ and $\alpha_{l2}^S$ to be directly related to the lateral forces that they support:

$$\begin{align*}
\alpha_{l1}^S &= e_S \frac{\tau_1 \cos \gamma}{r \sqrt{2} \cos \phi - \sin \phi} \left[ \begin{array}{c} 0c_1 \\
- \omega_2 \sin \phi c_2 \\
\omega_3 \cos \phi c_3 
\end{array} \right] \times \left[ \begin{array}{c} 0c_1 \\
- \cos \phi c_2 \\
- \sin \phi c_3 
\end{array} \right] \cdot e_1 \\
\alpha_{l2}^S &= e_S \frac{\tau_1 \cos \gamma}{r \sqrt{2} \cos \phi - \sin \phi} \left[ \begin{array}{c} \omega_3 \sin \phi c_1 \\
0c_2 \\
- \omega_2 \cos \phi c_3 
\end{array} \right] \times \left[ \begin{array}{c} \cos \phi c_1 \\
0c_2 \\
\sin \phi c_3 
\end{array} \right] \cdot e_2
\end{align*}$$

(21)

(22)

where $e_S$, called the lateral slip constant, has the unit 1/N. Its value is dependent on the geometry of the contact patch, material properties of the roller and the sphere, normal force, etc., $e_1$ and $-e_2$ ensure correct signs.

After some algebra, it can be shown that $\alpha_{l1}^S = \alpha_{l2}^S$. Therefore, let $\alpha_l = \alpha_{l1}^S = \alpha_{l2}^S$. Let the angle $\gamma_d$ describe the sphere’s actual axis of rotation. Whereas angle $\gamma$ describes the expected axis of rotation associated with the steering angle $\phi$, $\gamma_d$ describes the sphere’s actual axis of rotation. After some algebra, we can show $\gamma_d$ to be a function of $\phi$ and $\alpha_l$:

$$\gamma_d = \tan^{-1} \left( \frac{\sqrt{2} + \tan(\phi + \alpha_l)}{\sqrt{2} - \tan(\phi + \alpha_l)} \right)$$

(23)

Then, the modeled steering roller slip angle $\alpha_{SR}$ is simply the difference between $\gamma_d$ and $\gamma$:

$$\alpha_{SR} = \gamma - \gamma_d$$

(24)

### 3.3 Modeled Drive Roller Slip Angle

In the case of an ideal CVT, the velocity of drive Roller D1, $\omega_1$, is related to the parallel velocity $\omega_0$ and transmission angle $\gamma$: $\omega_1 = \omega_0 \cos \gamma$ (Eq. (13)). In practice, however, $\omega_1 \neq \omega_0 \cos \gamma$.

Let $\phi_1$ be the longitudinal slip ratio of Roller D1. Due to microslip incurred by D1, $\omega_1$ differs from $\omega_0 \cos \gamma$ by $\Delta \omega_1$. Then, from Eq. (18),

$$\omega_1 = \omega_0 \cos \gamma + \Delta \omega_1$$

Similarly, for drive Roller D2, $\omega_2$ differs from $\omega_0 \sin \gamma$ by $\Delta \omega_2$. Again, from Eq. (18),

$$\omega_2 = \omega_0 \sin \gamma + \Delta \omega_2$$

Given a load $\tau_1$ and a transmission angle $\gamma$, Roller D1 transmits a tractive force $-\tau_1, \sin \gamma/r$ on the surface of the sphere, where $r$ is the radius of the drive rollers. Similarly, D2 transmits a tractive force $-\tau_1, \cos \gamma/r$ on the surface of the sphere.

Let us model the ratios $\psi_1$ and $\psi_2$ to be linearly related to the tractive forces that the drive rollers transmit:

$$\psi_1 = \frac{\Delta \omega_1}{\omega_0 \cos \gamma}$$

(25)

$$\psi_2 = \frac{\Delta \omega_2}{\omega_0 \sin \gamma}$$

(26)

and

$$\psi_1 = e_D \frac{\tau_1 \sin \gamma}{r} \sgn(-\omega_1 \tau_1 \sin \gamma \cos \gamma)$$

(27)

$$\psi_2 = e_D \frac{\tau_1 \cos \gamma}{r} \sgn(\omega_2 \tau_1 \sin \gamma \cos \gamma)$$

(28)

where $e_D$, called the longitudinal slip constant, has the unit 1/N. The terms $\sgn(-\omega_1 \tau_1 \sin \gamma \cos \gamma)$ and $\sgn(\omega_2 \tau_1 \sin \gamma \cos \gamma)$ ensure correct signs.

Incorporating longitudinal slips of both drive rollers, we have

$$\omega_1 = \omega_0 \cos \gamma (1 + \psi_1)$$

(29)

$$\omega_2 = \omega_0 \sin \gamma (1 + \psi_2)$$

(30)

The actual velocities of the drive rollers, $\omega_1$ and $\omega_2$, may be expressed as a vector in $V$ space (Eq. (10)); let the angle $\gamma_m$ describe the direction of this vector. Then, drive roller slip angle $\alpha_{DR}$ can be found by subtracting $\gamma_d$ from $\gamma_m$:

$$\alpha_{DR} = \gamma_m - \gamma_d$$

(31)

### 3.4 Modeled Slip Angle

The modeled drive roller slip angle and the steering roller slip angle are summed to find the modeled CVT slip angle $\alpha$:

$$\alpha = \alpha_{SR} + \alpha_{DR}$$

(32)

### 4 Experimental Analysis

In this chapter, we describe the setup and experimental analysis of a CVT that is subject to various loads.

#### 4.1 Experimental Setup

Figure 11 shows the actual CVT used for experimental testing. The two drive rollers are labeled.
Only one of the two steering rollers is visible. A servomotor, used to generate CVT load, is directly coupled to each of the two drive rollers. The drive rollers are 82a durometer in-line skating wheels; they are 80 mm in diameter. The steering rollers are 84a durometer in-line skating wheels; they are 76 mm in diameter.

4.2 Testing Protocol. We are interested in finding the correlation between the CVT slip angle $\alpha$, velocity $\omega$, transmission angle $\gamma$, and load $\tau$. In our experiments, the transmission angles are set to values between −80 deg and +80 deg at 10 deg increments, and the CVT loads are set to values between −1.0 Nm and 1.0 Nm at 0.1 Nm increments.

The CVT load is considered to be positive if the following is true:

$$| (\omega_1 v_1 + \omega_2 v_2) \times (\gamma_1 v_1 + \gamma_2 v_2) | > 0$$

(33)

and negative if the following is true:

$$| (\omega_1 v_1 + \omega_2 v_2) \times (\gamma_1 v_1 + \gamma_2 v_2) | < 0$$

(34)

4.3 Experimental Results. With the first set of experiments, we want to determine if there exists a correlation between the CVT slip angle and the CVT velocity.

In Fig. 12, we show the measured CVT slip angles $\alpha$ versus the CVT velocity $\omega$ at various CVT loads with the transmission angle set to 45 deg. Figure 12 shows a lack of correlation between the CVT slip angle and CVT velocity and thus we may reasonably conclude that the measure of the slip angles is independent of the CVT velocity $\omega$. We may, therefore, carry out our experimental analysis of the CVT at any CVT velocity.

4.3.1 Measured Slip Angle. The measured velocities of both drive rollers may be expressed as vector $\omega$ in the coordinate frame $\Sigma$ (Fig. 7). Let the angle $\gamma$ describe the direction of this vector (Fig. 7). Then, the CVT slip angle $\alpha$ is the difference between the angle $\gamma$ and the CVT angle $\gamma$.

$$\alpha = \gamma - \gamma$$

(35)

Figure 13 shows the measured CVT slip angle versus the CVT angle at CVT loads from −1 Nm to 1 Nm in 0.1 Nm increments.

4.3.2 Measured Steering Roller Slip Angle. The measured steering roller slip angle $\alpha_{SR}$ can be found by measuring the difference between the measured axis of rotation angle and the commanded axis of rotation angle (also called the CVT angle) (Eq. (24)). We were able to determine the actual axis of rotational angle visually, using a grid of closely spaced wires placed a few millimeters above the sphere. Figure 14 shows the measured steering roller slip angle $\alpha_{SR}$.
ing roller slip angle $\alpha_{SR}$ versus the commanded CVT angle $\gamma$ at various CVT loads $\tau_L$.

### 4.3.3 Measured Drive Roller Slip Angle

We can find the measured drive roller slip angle $\alpha_{DR}$ using Eq. (31). Figure 15 shows the measured drive roller slip angle $\alpha_{DR}$.

Note that we have plotted the drive roller slip angles versus the actual transmission angles ($\gamma_d$) rather than the CVT angles ($\gamma$), thereby isolated the measurements of the physical effects at the rolling contacts between the sphere and the drive rollers.

### 4.4 Comparison Between Measured and Modeled Slips

Now that we found the slips experimentally, we can compare it to our slip models. Recall that our models have just two constants, $e_S$ and $e_D$, for us to define. First, we adjust $e_S$ so that our modeled steering roller slip angles closely match the measured steering roller slip angles. A comparison between Figs. 16 and 14 shows that our model accurately describes the physical effects at the contacts between the steering rollers and the CVT sphere. Next, we adjust $e_D$ so that the modeled drive roller slip angles closely match the measured drive roller slip angles. A comparison between Figs. 17 and 15 shows that our drive roller slip model closely describes the physical effects at the contacts between the drive rollers and the CVT sphere.

Finally, we sum the modeled slip angles $\alpha_{DR}$ and $\alpha_{SR}$ to find the modeled CVT slip angle $\alpha$. A comparison between Figs. 18 and 13 shows that our CVT slip model closely describes the various slips that take place at all four rolling contacts between the sphere and the rollers.

### 5 Suggested Design of the Continuously Variable Transmission

#### 5.1 Placement and Number of Rollers

Having determined the values of the constants $e_D$ and $e_S$, we can predict the slip angles of a CVT with a different roller configuration. Consider the box CVT that is discussed in Ref. [1].

The differences between the box CVT and the existing CVT are in the placements of the steering rollers and the addition of follower rollers.

The number and the arrangement of the drive rollers in the box CVT lessen the amount of slipover, which is incurred in the existing CVT. In the box CVT, the drive rollers (A and B in Fig. 19) are mechanically coupled to follower rollers (A' and B' in Fig. 19). Whereas in the existing CVT, tractive forces by the drive rollers were supported by just two drive rollers, tractive forces in the box CVT are supported by four drive rollers, thereby essentially halving the drive roller slip angle.

Another difference between the two CVTs is that the box CVT employs additional steering rollers that are positioned at different positions around the sphere. This setup significantly lessens the amount of slip incurred by the steering rollers as we will show later.

Figure 19 shows the suggested design of the CVT. The pro
posed CVT employs a pair of Rollers A and A’ that are mechanically coupled. These two rollers transmit motions and torques to a single rotational joint of a mechanical system, such as a robot. Rollers B and B’ are also mechanically coupled; they both transmit motions and torques to a second rotational joint. Two sets of a pair of steering rollers (each separated by a distance d) are located on opposite ends of the sphere.

Let us assume that the box CVT employs the same types of rollers and sphere as the existing CVT. Let us also assume that the preload force is the same as it was for our experiment. We can then predict the slip angles \( \alpha, \alpha_{SR}, \) and \( \alpha_{DR} \) for the box CVT (with \( \varepsilon_D = 0.0038 \ N^{-1} \) and \( \varepsilon_S = 0.0049 \ N^{-1} \)).

Figure 20 shows the steering roller slip angles and the drive roller slip angles for the existing CVT (Fig. 2) and Fig. 21 shows the slip angles for the box CVT. A comparison between Figs. 20 and 21 shows that slip angles are smaller in the box CVT than in the existing CVT.

The existing CVT employs two steering rollers to constrain the sphere to rotate about a particular axis of rotation, whereas the box CVT employs four steering rollers to accomplish the same task. A load across the CVT creates a moment about an axis that must be balanced by the tractive forces at the rolling contacts between the sphere and the steering rollers. Since the box CVT employs more steering rollers than the existing CVT, they are required to support lesser tractive forces.

One other improvement over the existing CVT is the position of the steering rollers about the surface of the sphere. In the box CVT, the steering rollers are positioned about the sphere such that their tractive forces produce the maximal moment about the sphere.

6 Conclusion

In this paper, we presented experimental results that showed that loads across the CVT cause the velocities of the CVT joints to differ from the intended velocities. Our experimental results showed that for a particular transmission angle, the difference between the actual velocity ratio and the ideal velocity ratio is linearly related to the load applied across the CVT. We observed that the cause of this difference is microslip at the rolling contacts between the sphere and the rollers. The CVT’s steering rollers incurred lateral slip, resulting in the sphere’s axis of rotation deviating from the intended tilt angle. The CVT’s drive rollers incurred longitudinal slip due to tractive forces that they had to support.

The lateral and longitudinal slips incurred by the steering and drive rollers were characterized by two constants \( \varepsilon_D \) and \( \varepsilon_S \), each...
relating the measures of the slips to the tractive forces supported across the rolling contacts. After having found the values of these two constants, we showed that the proposed box CVT would perform superior to the existing CVT.

References


