A Controller For Simulating Freedom Of Motion For A Cobot
Carl A. Moore, Michael A. Peshkin, J. Edward Colgate
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A CONTROLLER FOR SIMULATING FREEDOM OF MOTION FOR A COBOT

Carl A. Moore — *Mechanical Engineering, Northwestern University, Evanston, IL 60208, camoore@northwestern.edu*

Michael A. Peshkin — *Mechanical Engineering, Northwestern University, Evanston, IL 60208, peshkin@northwestern.edu*

J. Edward Colgate — *Mechanical Engineering, Northwestern University, Evanston, IL 60208, colgate@northwestern.edu*

Abstract: A method is presented for creating a controller to simulate free motion for a cobot. A cobot is a class of mechanically passive robotic devices, intended for direct physical collaboration with a human operator (Colgate, et al., 1996a). One common cobot application involves a person using the cobot's endpoint to probe a virtual surface. This task requires that the user have freedom of motion while not in contact with the virtual surface. Because a cobot has only one degree of freedom at any given time, free motion must be simulated by re-directing this one-degree of freedom to coincide with the user's desired motion direction. A technique for creating the free motion controller for a cobot is developed and simulated on a three revolute arm-like cobot.

1. INTRODUCTION

Unlike traditional robots that use motor actuated joints, cobots employ nonholonomic elements called continuously variable transmissions (CVTs) (Colgate, et al., 1996b). Each CVT holds the speed of two joints in a computer-controlled variable ratio. With each pair of joints coupled to a CVT, the cobot's allowed motion direction is mechanically confined to 1 degree of freedom. However the CVT's transmission ratios can be changed or "steered" in real time giving the cobot the illusion of having a full range of motion.

Cobots have two primary modes of operation, free motion (caster) mode, and constraint mode. In free motion mode the cobot operates as follows. A user applies a force by pushing on a force-sensor-equipped handle or endpoint. Force components parallel to the allowed motion direction simply produce movement in that direction. Steering the CVT transmission ratios such that the allowed direction becomes parallel to the user-input force direction actively minimizes force components perpendicular to the allowed direction. Using this technique the cobot reacts just like a chair on casters, it instantaneously reacts to all user forces allowing the user to freely position the cobot within its workspace.

In constraint mode virtual walls or surfaces are constructed in the cobot's workspace (Gillespie, et al., 1996). If the user brings the cobot into contact with one of these constraints, the computer ceases to steer the CVTs such that all perpendicular forces are minimized. Instead the CVTs are steered such that the allowed motion direction remains parallel to the constraint surface. Any user forces that would cause the cobot to penetrate the wall are ignored. Forces that would pull the cobot off of the wall and back into the free space are interpreted as in free motion mode.

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When a user applies forces into a constraint created by a traditional robotic device, actuators are used to produce equal and opposite forces. Invariably stability problems arise in the control of these systems, and large actuators must be used to produce constraints that seem weak in comparison (Colgate and Brown, 1994). Cobots represent a superior solution because their constraints are a function of the intrinsic mechanics of CVTs. No power is needed to maintain arbitrarily oriented constraints that are smooth, strong, and stable (Book, et al., 1996; Delnondedieu and Troccas, 1995).

There are other modes of cobot operation, but invariably the cobot will be required to allow the user complete freedom of motion in some area of its workspace. For this reason free motion is the most fundamental of cobot modes.

2. FREE MOTION CONTROL

During free motion, the goal of control is to make the cobot appear transparent to the user by permitting any desired motion (Wannasuphoprasit, et al., 1997). Consider the simplest of cobots, a unicycle cobot (Figure 1) consisting of a rolling wheel in contact with a planar working surface.

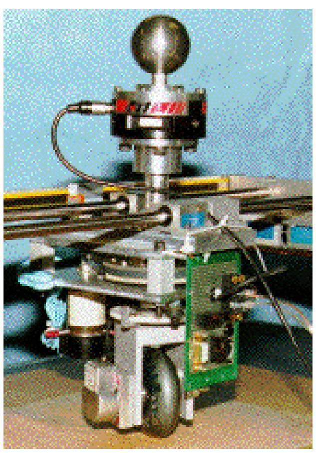

Figure 1. Unicycle cobot.

The wheel is attached to an upright handle, and a motor is used to steer the wheel about the handle. The motor cannot cause the wheel to roll; it can only change the wheel's rolling direction. A force sensor on the handle measures user forces. In effect the wheel is a CVT in that couples the x and y axis velocities in a continuously variable proportion according to the tangent of the steering angle θ or:

$$\tan\theta = \frac{\dot{y}}{\dot{x}} \qquad (1)$$

In the unicycle, free motion is accomplished by perceptually eliminating the wheel and making the cobot feel like a point mass. As a point mass, any user applied force on a cobot results in an instantaneous acceleration whose magnitude and direction are in fixed proportion. In practice, the unicycle's allowed motion direction is determined by the direction of the steered wheel. When this direction is parallel to the user's force direction the cobot does behave like a point mass, and the resulting acceleration $a_{\|}$ is

$$a_{\|} = \frac{F_{\|}}{M} \qquad (2)$$

where $F_{\|}$ denotes a force parallel to the current rolling direction.

A force $F_{\perp}$ that is perpendicular to the allowed motion direction must also produce an acceleration $a_{\perp} = F_{\perp}/M$. To achieve this acceleration, the wheel is steered to change the allowed direction. The speed at which the wheel is steered can be determined by noting that a point mass moving in a circular path with speed u and mass M has a centripetal force $F_{\perp} = d\theta/dt\ uM$, where $d\theta/dt$ is the rate of change of the angle between the x-axis and the force $F_{\perp}$. Therefore, a prescription for the steering speed of the wheel such that the cobot behaves like a point mass is,

$$\frac{d\theta}{dt} = \frac{F_{\perp}}{uM} \qquad (3)$$

One could also have arrived at the above steering velocity equation by differentiating with respect to time the unicycle's transmission equation (Equation 1) relating task space velocities to the steering wheel angle θ.

$$\frac{d(\tan\theta)}{dt} = \sec^2\theta\dot{\theta} = \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2} \qquad (4)$$

where,

$$u = \sqrt{\dot{x}^2 + \dot{y}^2}$$
$$\ddot{x}M = F_{\|}c\theta - F_{\perp}s\theta$$
$$\ddot{y}M = F_{\|}s\theta + F_{\perp}s\theta$$

After simplification the steering velocity is

$$\dot{\theta} = \frac{F_{II}(c\theta s\theta - c\theta s\theta) + F_{\perp}(c^2\theta + s^2\theta)}{uM} = \frac{F_{\perp}}{uM} \qquad (5)$$

which is identical to the original Equation 3.

The F_{II} argument is zero accounting for the fact that no change in steering angle θ is required for the unicycle to continue rolling in the direction of F_{II}.

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It is also important to note that Equations 2,3, and 5 use the unicycle's actual mass, M, because isotropic point mass behavior is desired. In other words, the goal was for the unicycle to feel like a mass of M in both the perpendicular and parallel directions. Since the unicycle's wheel is not powered in its rolling or parallel direction, there is no control over what it feels like in that direction. However, replacing this mass with a different value (i.e.: $M_\perp$), makes the unicycle feel like it has mass $M_\perp$ in the perpendicular direction:

$$\dot{\theta} = \frac{F_\perp}{uM_\perp} \tag{6}$$

The perpendicular force $F_\perp$ required to change the unicycle's heading will be magnified according to the selection of $M_\perp$.

It was rather straightforward to develop the steering speed equation for the simple unicycle cobot that makes it behave like a point mass. Now our attention turns to the more complicated 3dof arm cobot.

3. FREE MOTION FOR 3-REVOLUTE ARM COBOT

Figure 2 is a drawing of the arm cobot's four-link parallelogram manipulator and CVTs. The CVTs are called rotational CVTs, and are used with revolute jointed cobots (Gopalswamy, et al., 1992; Moor, et al., 1999).

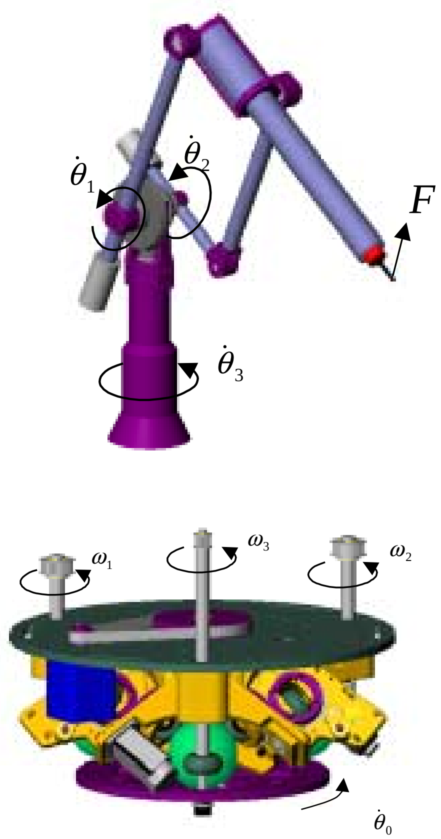

Figure 2. Manipulator and 3 CVTs in parallel.

The angular velocity of each joint is connected to a drive roller of one of the three CVTs using timing belts that are not shown. The relationship between the drive roller angular velocities $d\omega_i$ and the joints velocities $d\theta_i$ is

$$\dot{\omega}_i = t_r \dot{\theta}_i \tag{7}$$

where t_r is a constant timing belt pulley ratio.

A common wheel, the "power wheel", is in rolling contact with each of the CVTs, meaning that the CVTs are connected in parallel. The kinematics of the rotational CVT is not reproduced herein was described by Moore (1997). The transmission ratio equation for this arrangement of CVTs is

$$\mathbf{T} = \tan \gamma_{1-3} = \frac{\omega_{1-3} r_d}{\dot{\theta}_0 r_p} \tag{8}$$

where r_d and r_p are the radius of the power plate and drive roller respectively, and the angles $\gamma_{1\text{-}3}$ are the steering angles for the spherical CVTs (Moore, 1997).

In the passive scenario where the power wheel is allowed to rotate freely, its angular velocity is proportional to the joint velocities,

$$\dot{\theta}_o = \frac{t_r k_r \left|\dot{\theta}\right|^2}{\dot{\theta}^{\mathrm{T}} \mathbf{T}} \tag{9}$$

where k_r is the ratio r_d/r_p.

The parallel arrangement of CVTs results in a redundancy such that the CVT transmission ratios ($T_{1\text{-}3}$) can not be determined independently of the power wheel velocity. So, a variable k is defined that relates the speed of the power wheel to the speed of the cobot's endpoint:

$$\dot{\theta}_0 r_p = k\left|\dot{\mathbf{x}}\right| \tag{10}$$

Now the CVT transmission ratios can be solved as a function of k, the one allowed degree of freedom in task space $\mathbf{v}/|\mathbf{v}|$, and the Jacobian $\mathbf{J}$ relating joint space velocities to task space velocities,

$$\mathbf{T} = \tan \gamma_i = \frac{t_r r_d \mathbf{J}^{-1}}{k} \frac{\dot{\mathbf{x}}}{\left|\dot{\mathbf{x}}\right|} \tag{11}$$

4. ARM COBOT FREE MOTION CONTROL

For the arm to simulate freedom of motion, the steering angles $\gamma_{1\text{-}3}$ must be updated according to steering velocities $d\gamma_{1\text{-}3}/dt$. As with the unicycle, these velocities can be found by differentiating the vector of

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CVT transmission ratios, Equation 8. However, it might prove more intuitive to differentiate these equations in task space; therefore, using Equation 11.

$$\frac{d\mathbf{T}}{dt} = \sec^2 \gamma\dot{\gamma} = t_r r_d \frac{\mathbf{J}^{-1}\left[\left(\ddot{\mathbf{x}} - \dot{\mathbf{J}}\mathbf{J}^{-1}\dot{\mathbf{x}}\right)k|\dot{\mathbf{x}}| - \dot{\mathbf{x}}\left(\dot{k}|\dot{\mathbf{x}}| + k\frac{\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|}\cdot\ddot{\mathbf{x}}\right)\right]}{\left(k|\dot{\mathbf{x}}|\right)^2} \tag{12}$$

where $\dot{\mathbf{J}}$ is the Hessian,

$$\left\{\frac{\partial \mathbf{J}}{\partial \theta}\right\}_i = \sum_{j=1}^{3}\left[\sum_{k=1}^{3}\frac{\partial \mathbf{J}_{(ij)}}{\partial \theta_{(k)}}\right] \tag{13}$$

and the derivative of the task space velocity magnitude is

$$\frac{d}{dt}|\dot{\mathbf{x}}| = \frac{\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|}\cdot\ddot{\mathbf{x}} \tag{14}$$

The derivative of k, is a feedback term:

$$\dot{k} = \frac{k_d - k}{dt} \tag{15}$$

where k_d is the desired value of k.

Remembering from Equation 5, that only forces perpendicular to the allowed direction impact the CVT steering speeds, use the following substitution for the task space acceleration terms:

$$\ddot{\mathbf{x}} = \mathbf{F}_\perp \mathbf{M}_\perp^{-1} \tag{16}$$

where $\mathbf{M}_\perp^{-1}$ is the inverse of the desired arm mass matrix in the perpendicular direction. Again, without adding power we can not alter the perceived mass in the parallel direction.

Substituting Equation 16 into Equation 14 and noting that the perpendicular force is always perpendicular to the instantaneous velocity direction, their dot product in Equation 12 is also always zero:

$$\frac{\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|}\cdot\mathbf{F}_\perp\mathbf{M}_\perp^{-1} = 0 \tag{17}$$

After making the above substitutions, a simpler form of the CVT steering velocities is obtained.

$$\dot{\gamma} = t_r r_d \frac{\mathbf{J}^{-1}\left[\left(\mathbf{F}_\perp\mathbf{M}_\perp^{-1} - \dot{\mathbf{J}}\mathbf{J}^{-1}\dot{\mathbf{x}}\right)k - \dot{\mathbf{x}}\dot{k}\right]}{|\dot{\mathbf{x}}|\left(k\sec\gamma\right)^2} \tag{18}$$

As with the unicycle, these steering velocities are undefined for zero endpoint velocity. When the endpoint velocity is near zero, the controller will command near maximum steering velocities so that the allowed motion direction is brought parallel to the desired motion direction instantaneously.

5. FREE MOTION CONTROLLER

The free motion controller operates in the following sequence:

1) Desired user motion is measured using an intent sensor, which in our case is a force sensor.
2) Computer calculates the necessary steering velocities using Equation 18.
3) These velocities are commanded to the CVT steering motors.

There are many ways to categorize a good free motion controller. Generally, a good free motion controller should result in the cobot feeling like it has mass $M_\perp$ in the perpendicular direction as it is propelled by the user. Redundant cobots, such as a power assist cobot with parallel connected CVTs, have many CVT setting solutions for a given allowed direction in task space. For these cobots, the controller must pick the "best" steering velocities from the solution set. In the particular case of the arm, the problem is addressed by introducing the variable k that is the ratio of power wheel speed to task space speed. The steering controller then has the added job of keeping this ratio equal to a desired value.

Figure 3 is a block diagram for the free motion controller.

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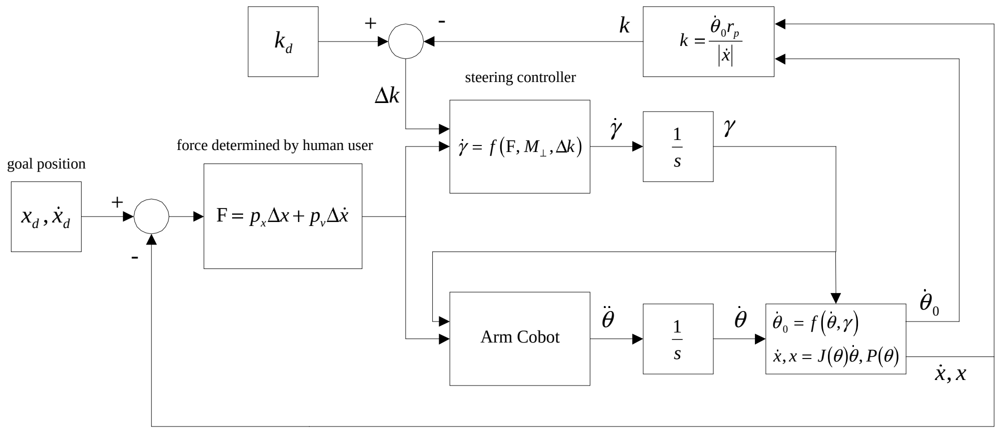

Figure 3. Block diagram of the free motion steering controller.

6. SIMULATION OF ARM COBOT FREE MOTION

Figure 4 is a photo of the arm cobot. The links are not attached. The origin of the three joints is 1.5m above the floor. The arm cobot has a maximum reach of about 80cm.

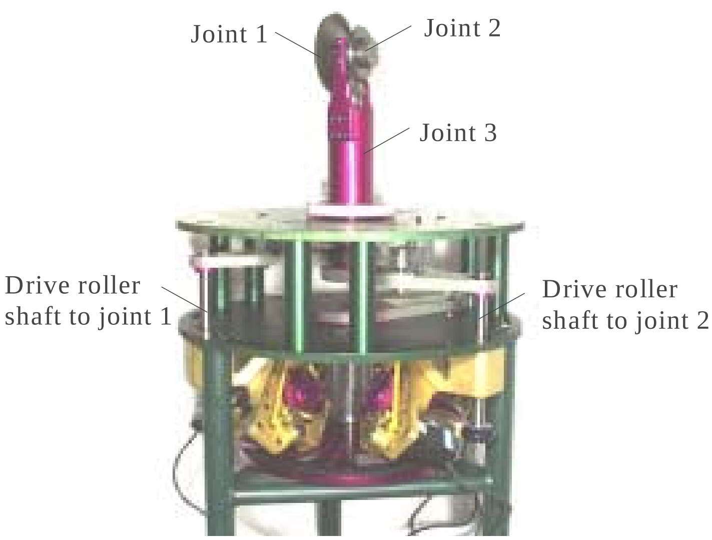

Figure 4. Arm Cobot.

In the simulation, there is a planned path and planned velocity represented by a set of points x and their derivatives $(x_{i+1}-x_i)/dt$. The forces applied to the endpoint are proportional to the error in position and velocity. In actual free motion there would not be a planned path because any user desired motion would be followed; a planned path is added here only to provide a means of calculating realistic forces to apply to the cobot's endpoint.

Figure 5 shows simulation results in task space for a straight-line path in the yz plane.

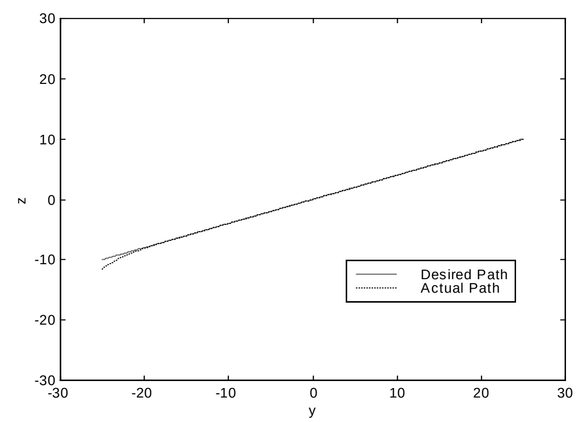

Figure 5. Arm cobot following straight-line path.

A more extravagant task space motion is simulated in Figure 6. The path followed is a figure eight "8" in the xz plane.

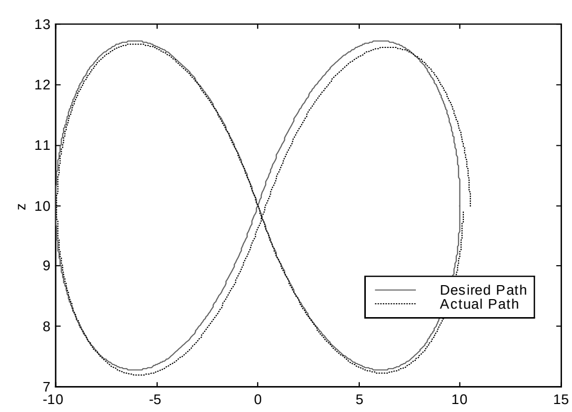

Figure 6. Arm cobot following a figure "8" path.

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The corresponding controller produced CVT steering velocities and the resulting steering angles are shown in Figure 7.

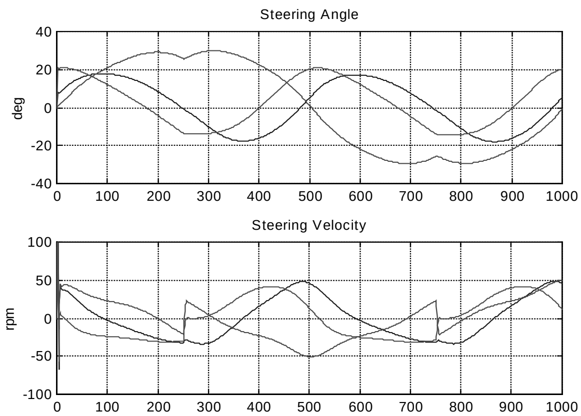

Figure 7. CVT steering angles and velocities.

As noted, it is also important for the free steering controller to maintain the desired ratio k of power wheel velocity to endpoint velocity. For this simulation the desired k was 1. Figure 8 is a plot of k during the simulation.

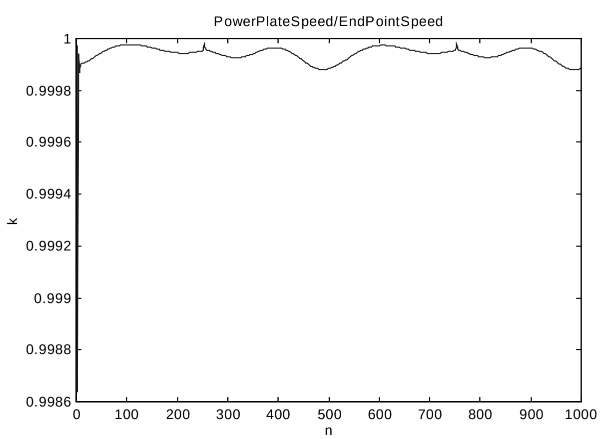

Figure 8. Ratio k.

CONCLUSION

The steering velocities computed by the controller produce the desired free endpoint motion while maintaining the correct ratio between the endpoint and power wheel speeds. The steering velocities were developed by simple differentiation of the CVT transmission ratios resulting in velocity equations that completely described the system under free motion control. Some complexity does arise when transforming the original joint space equations into task space, but these transformations are no more complicated than what is common to traditional robotics. There is reason to believe that this method of creating a free motion controller will be applicable to other cobot architectures.

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