A Three Revolute Cobot Using CVTs in Parallel
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A Three Revolute Cobot Using CVTs in Parallel

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ABSTRACT

Cobots are capable of producing virtual surfaces of high quality, using mechanical transmission elements as their basic element in place of conventional motors. Most cobots built to date have used steerable wheels as their transmission elements. We describe how continuously variable transmissions (CVTs) can be used in this capacity for a cobot with revolute joints.

The design of an “arm-like” cobot with a three-dimensional workspace is described. This cobot can implement virtual surfaces and other effects in a spherical workspace approximately 1.5 meters in diameter. Novel elements of this cobot include the use of a power wheel that couples three CVTs that are connected in parallel.

INTRODUCTION

Several robotic devices have been proposed for the purpose of creating programmable constraints and virtual surfaces. One such device by Book et. al. [1], called P-TER (Passive Trajectory Enhancing Robot), is a 2-degree of freedom (dof) manipulator designed to guide its end effector along a desired path while being pushed by the user. Clutches are used to vary the coupling between the two major links of the device, while brakes are used to remove energy from the links. Delnondedieu and Troccaz [2] have developed another device, called PADyC (Passive Arm with DYnamic Constraints), intended for guided execution of potentially complex surgical strategies. The prototype system has 2 dof and uses 2 each of a motor, clutch, and free wheel to dynamically constrain each joint.

Neither of these devices is able to provide arbitrarily oriented, smooth, hard virtual surfaces.

SCOOTER COBOT

To illustrate how cobots provide smooth, hard virtual surfaces, we use the example of Scooter (shown in Figure 1), a cobot with a three-dimensional workspace (x, y, θ) [3]. Three small motors are used to steer the wheels of which two are visible in Fig. 1. The motors cannot cause the wheels to roll; they can only change the wheels’ rolling direction. A force sensor on the center post handle measures forces applied by the user.

A cobot’s two simplest modes of operation are free mode (in which the user can move the cobot freely in (x.y.θ)-space) and virtual surface mode (in which only motion along a virtual constraint is allowed).

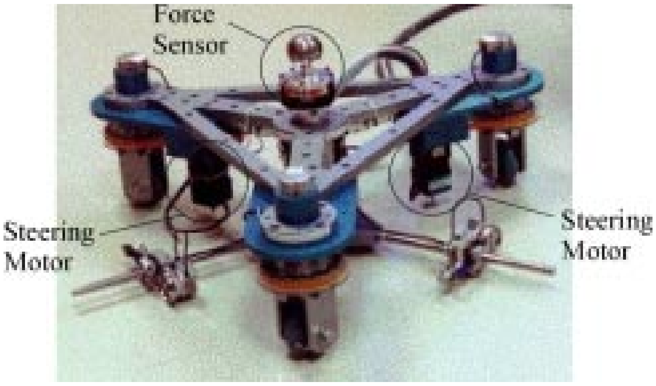

Figure 1. Scooter three wheeled cobot.

Free Mode:

In free mode Scooter operates as if it were supported by casters, like those on an office chair, which permit any desired motion direction. Unlike casters whose shafts are off center, Scooter’s wheels are on straight-up shafts and are steered using motors. When the user applies a force to Scooter by pushing on the center handle, the computer monitors the force perpendicular to Scooter’s rolling direction and attempts to minimize it by changing Scooter’s rolling direction. Scooter’s rolling direction is described by a center of rotation (COR). If the COR lies directly in the center of Scooter, the only allowed motion is rotation about the handle. If the COR is infinitely far away, corresponding to all wheel axes being parallel, then Scooter will follow a straight line. Therefore, in free mode, the computer monitors the user’s forces, determines the required COR, and turns the wheels to allow that motion.

Virtual Surface Mode:

In virtual surface mode a cobot filters the user’s motion. If the user brings Scooter up to a programmed virtual surface, the computer ceases to steer the wheels in a direction that minimizes the perpendicular force. Instead, the wheels are steered such that the allowed motion is tangent to the surface. The computer does continue to monitor the user-applied forces. Forces that would cause Scooter to penetrate the surface are ignored. Forces that would bring Scooter off of the surface and back into the free space are interpreted as before in free mode, and Scooter again behaves as if it were on casters.

When a cobot is in contact with a virtual surface or constraint, it is possible for the user to apply a force into the constraint that is large enough to cause the constraint to collapse. The strength of the virtual constraint is related to the mechanism by which the cobot resists perpendicular forces. With Scooter, coulomb friction forces between the steered wheels and the working planar surface resist forces applied against the constraint. If the applied force becomes greater than the friction force, the virtual surface crumbles and the cobot enters the restricted area.

ROTATIONAL CVT

Scooter is restricted to a three-dimensional planar workspace because its virtual surface behavior relies on the presence of a flat working surface on which to roll. Revolute arm-like architectures have proven very versatile for robots, and so we now address the problem of creating an arm-like cobot with revolute joints.

The role of the steered wheels in Scooter is to establish a mechanically enforced ratio between the x-velocity and the y-velocity of the steering axis of each wheel. That ratio, v_y/v_x, is given by α, the steering angle of the wheel, which is under computer control. This principle may be considered obvious for a wheel, but it lies at the heart of cobots that have planar workspaces, like Scooter.

To extend cobots to workspaces typical of revolute jointed robots, we require a mechanical element whose function is analogous to that of the wheel in scooter. For revolute joints the mechanically enforced ratio is between two angular velocities, ω_1 and ω_2, rather than two linear velocities as in scooter. Also, the ratio ω_2/ω_1, which is enforced mechanically, must be adjustable under computer control just as the angle of each of Scooter's wheels.

The requirements above call for a continuously variable transmission, or CVT: a device which couples two angular velocities according to any adjustable ratio. A drawing of such a CVT is shown in Figure 2.

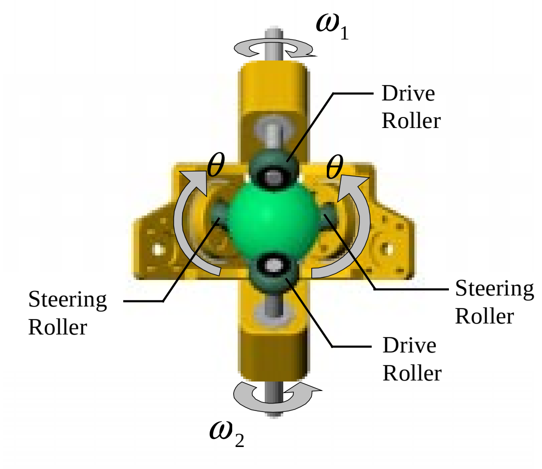

Figure 2. Rotational CVT.

A device with similar components has been created by Sordalen et. al. [4,5] for the study of nonholonomic manipulator control.

The CVT consist of a sphere caged by four rollers. The rollers are arranged at the corners of a stretched tetrahedron so that the angle subtended by the contact points is 90° (a regular tetrahedron would have angles of 108°). The two rollers with angular velocities labeled ω_1 and ω_2 are called drive rollers; their orientation is fixed. The other two rollers are called steering rollers; their orientation is measured by an angle θ. The axes of the two drive rollers share a plane that also includes the center of the sphere. Rolling constraints dictate that the sphere's rotational axis must lie in the plane of the drive rollers and go through the non translating center of the sphere. Figure 3 is a diagram of the plane that contains both drive roller axes and the sphere's rotational axis. I use the angle γ to denote the displacement of the sphere's axis from drive roller two.

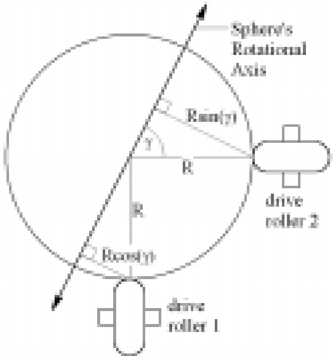

Figure 3. Plane containing sphere's rotational axis.

The steering rollers impose additional rolling constraints on the CVT such that the angle γ is a function the steering roller setting θ,

$$\gamma = \tan^{-1}\left(\frac{\sqrt{2}}{2}\tan\theta\right) - 45°. \qquad (1)$$

The CVT transmission ratio T can be interpreted as follows. From Figure 3, $R\cos(\gamma)$ and $R\sin(\gamma)$ are the radius of the paths that drive roller 1 and 2 follow about the sphere as it rotates. Assuming the sphere (radius R) has an angular velocity Ω, and the drive rollers (radius r) do not slip on the sphere, the angular velocities of the drive rollers can be written as

$$\omega_1 = \frac{\Omega R\cos\gamma}{r} \quad \omega_2 = \frac{\Omega R\sin\gamma}{r}. \qquad (2)$$

The transmission ratio is the ratio of drive roller angular velocities or

$$\mathrm{T} = \frac{\omega_2}{\omega_1} = \tan\gamma . \qquad (3)$$

As the steering rollers are turned from -90° to +90°, the transmission ratio ω_2/ω_1 assumes the full range of values from $-\infty$ to $+\infty$ [6]. Of course friction prevents the CVT from achieving an infinite transmission ratio. In practice a maximum transmission ratio of 20:1 is common.

SERIAL COBOT

Figure 4 is a drawing of a four link parallelogram arm that will constitute the manipulator of the arm cobot. The arm has three joints whose angular velocities are held in computer controlled ratios by three CVTs and is equipped with a force sensor to measure user intent.

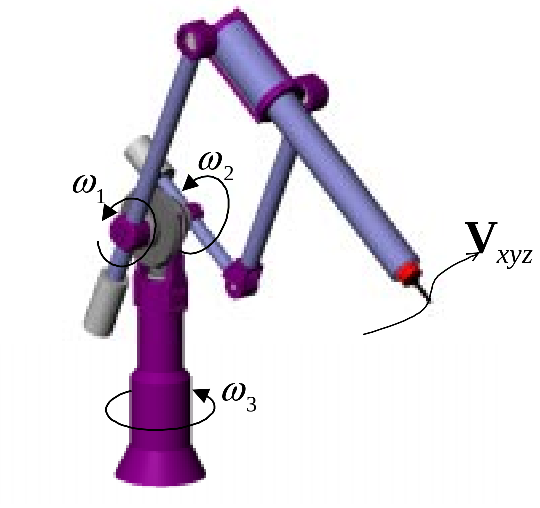

Figure 4. Parallelogram link architecture.

We chose to use CVTs connected in parallel to control the arm's joints. In this configuration the angular velocity of each joint is coupled to a separate drive roller and the remaining three drive rollers are tied to a common shaft. A three joint schematic of such an arrangement is shown in Figure 5. The steering rollers and support frames are not shown.

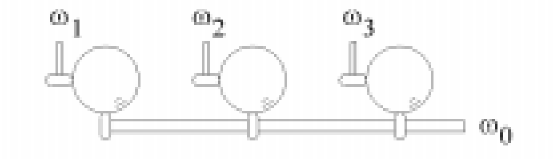

Figure 5. Three CVTs in parallel.

In this configuration, a CVT transmission ratio T_i relates the drive roller angular velocity ω_i to the common shaft velocity ω_0

$$\mathrm{T}_i = \frac{\omega_i}{\omega_0}, \qquad (4)$$

For each CVT, the transmission ratio is proportional to the angular speed of its joint ω_i. Therefore, a near zero joint speed requires merely that its own CVT have a transmission ratio which is also nearly zero. So, the speed ratio between any two joints connected by CVTs in parallel can assume the full range of values ($-\infty$ to $+\infty$) with finite CVT transmission ratios.

The three transmission ratios determine the direction of the allowed angular velocity vector in joint space. The magnitude of this vector is scaled by the velocity of the common shaft ω_0. The task space velocity vector is similarly constrained through the arm's Jacobian such that at any given time the manipulator has one degree of freedom. By steering the CVT transmission ratios, the allowed motion vector in task space can be pointed in any direction. When this is done in real time, the arm appears to have 3 degrees of freedom.

If the common shaft is allowed to rotate freely then its angular velocity is proportional to the speed at which the user manipulates the arm's end point. Using the user imposed velocity $\mathbf{V}_u$, the inverse Jacobian $\mathbf{J}^{-1}$, and a vector of CVT transmission ratios $\mathbf{T}$, the angular velocity of the freely rotating common shaft is

$$\omega_0 = \frac{\mathrm{T}^{\mathrm{T}}\mathbf{J}^{-1}\mathrm{V}_{\mathrm{u}}}{\mathrm{T}^{\mathrm{T}}\mathrm{T}} . \qquad (5)$$

If the common shaft is not allowed to rotate freely but is connected to a motor then the task space velocity $\mathbf{V}_{xzy}$ of the arm's end point is related to the motor speed ω_0 by,

$$\mathrm{V}_{xyz} = \mathbf{J}\mathrm{T}\omega_0 . \qquad (6)$$

The same ideas hold for endpoint forces. Each joint torque τ_i is a product of the common shaft torque τ_0 and the inverse of its CVT transmission ratio such that the force $\mathbf{F}_{xyz}$ reflected to the endpoint by the motor is

$$\mathrm{F}_{xyz} = \mathbf{J}^{\mathrm{T}^{-1}} \begin{bmatrix} 1/\mathrm{T}_1 \\ 1/\mathrm{T}_2 \\ 1/\mathrm{T}_3 \end{bmatrix} \tau_0 . \qquad (7)$$

The noteworthy result of connecting CVTs in parallel is that regardless of the dimension of the cobot's task space, one motor can produces an endpoint force and speed that is parallel to the allowed motion direction.

The introduction of a motor negates the inherent safety of a passive cobot, but facilitates the accomplishment of important goals. When not in contact with a virtual surface a cobot should feel nearly transparent to the user. However, when we designed cobots with larger gear ratios to create harder constraints, the increased friction reflected to the user became a problem. Also, we desired to make cobots more responsive by magnifying the user's force at low speed. Both of these goals were conveniently accomplished by the addition of a motor.

Another interesting characteristic of CVTs in parallel is their ability to assume any angular velocity ratio without changing the steering roller settings. If the transmission ratios of each CVT are set to infinity, the common shaft has zero angular velocity and each joint can rotate freely without respect to any other joint. The only other time that the speed of the common shaft is zero is when the speed of all joints is zero.

ARM COBOT WITH POWER ASSIST

Figure 6 is a picture of the arm cobot built to date. The four link arm (similar to the drawing in Figure 4) has not been attached.

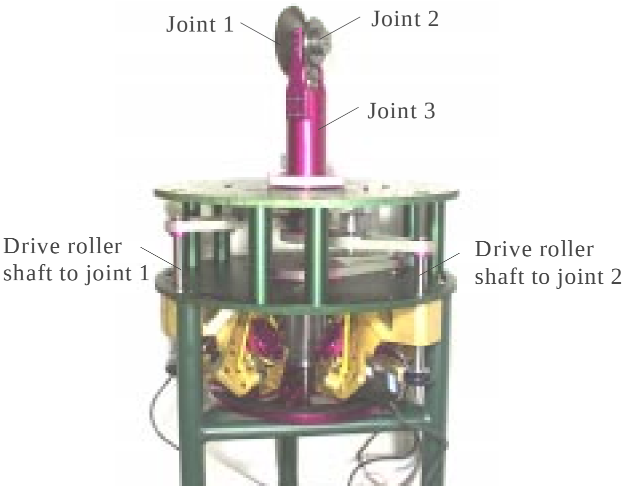

Figure 6. Arm Cobot.

It was decided early on that the cobot's CVTs would remain stationary (grounded) during motion of the arm. Connecting each CVT to ground lowers the complexity of the design and decouples the mass of the CVT subsystems from the arm's dynamics. As noted earlier, CVTs in parallel have the rotation of one of their drive rollers coupled to a common shaft. In this design the rotations of each CVT are coupled together using a common wheel as shown in Figure 7.

This common wheel or "power wheel" is in rolling contact with the central sphere of each CVT and can be driven using a 180 watt brushless servo motor (motor located in the bottom center of the figure). The power wheel is made from an aluminum plate that has a neoprene rubber running surface on one side. The power wheel has a diameter of 36.8cm, and its running surface contacts the CVTs 16.5cm from the center post.

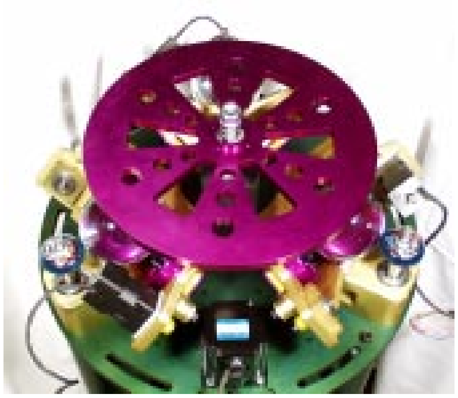

Figure 7. 3 CVTs, power wheel, and assist motor.

There are many benefits to this symmetric arrangement of CVTs. It permits a single spring on the power wheel's axle to apply an equal preload force to all CVTs. Also, in this arrangement, the drive roller shafts are parallel to their joint shaft axes allowing power transfer between the two with zero backlash timing belts. The design couples the joint rotations to three concentric shafts. Two sets of bevel gears connect the two non-vertical joint axles (joints 1 and 2) to the two innermost shafts. The third joint axle (joint 3) does not require bevel gears because its axis of rotation is already parallel to the CVTs' drive roller shafts.

The CVT (Figure 8), has a 10.16cm diameter acrylic sphere. The drive and steering rollers are 57mm 85A roller blade wheels. The steering wheels are enclosed in "steering hubs". Equal and opposite rotation of each hub is ensured by bevel gears that are synchronized to each hub through timing belts. There is a 45-watt steering motor with encoder on each CVT assembly.

Figure 8. CVT

Figure 9. CVT - back view.

The coulomb friction forces that exist between the rolling elements of the CVTs determine the force of constraint that the arm cobot can display. Assuming that the coefficient of static friction between the CVT's rolling elements (polyurethane wheels on acrylic sphere) is $\mu_s = 0.8$, a drive roller radius of 2.85cm, and a normal force of 7.00Kg, the resulting maximum torque that the drive rollers can resist is 16.0Kg-cm. With a gear ratio of 6:1 between the drive rollers and the joints, the maximum sustainable joint torque is 96.0kg-cm.

Figure 10 uses force ellipses to display the expected static force characteristics for the arm [7]. The major (minor) axis of each ellipse represents the maximum (minimum) endpoint force that can be resisted in the direction of the axis at that point in the workspace. The largest force that can be supported at a position is recorded in kilograms next to each ellipse.

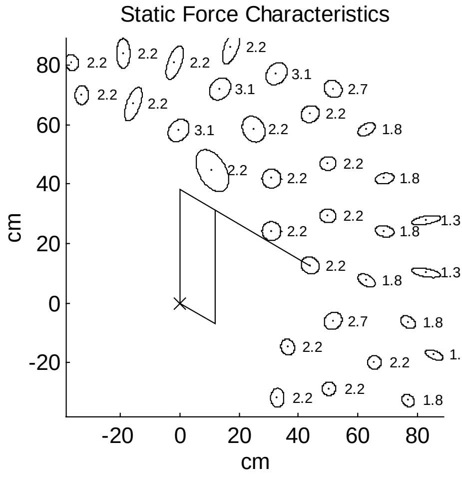

Figure 10. Static force characteristics.

The arm cobot's links will be constructed from graphite due to its high bending strength (E = 96MPa) and low weight ($\rho = 1.66\ g/cm^3$). The largest diameter link will have a 8.26cm OD, and the arm's reach will be over 76cm. With the support stand attached, the origin of the three joint rotations is approximately 1.5m above the floor. Figure 11 is a diagram of the cobot's anticipated workspace. This planar workspace becomes a volume when revolved about the z axis.

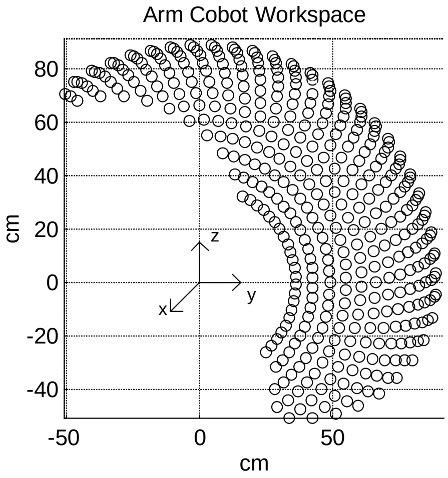

Figure 11. Arm workspace.

The parallelogram configuration permits the links to be counter-balanced against gravity for all arm configurations by two counter weights attached to joints 1 and 2 on the opposite side of the links [8]. The moments provided by the counter weights will be approximately 67.0Kg-cm and 39.0Kg-cm for joints 1 and 2 respectively.

CONTRIBUTION OF DESIGN

The parallelogram arm cobot will be the first 3R cobot. Its ability to move in traditional x,y,z 3-space opens an entire new class of tasks to cobotic solutions. The addition of power assist to the traditional passive cobotic model will result in a cobot that can reduce or magnify the inertia that is reflected to the user making larger cobots or cobots with large transmission ratios possible.

The powered arm will be able to perform tasks autonomously like a traditional robot while remaining backdrivable. A backdrivable arm is attractive to persons that want a powered manipulator that can also be easily positioned by hand, such as those interested in robot-assisted surgery.

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