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Mechanically Implementable Accommodation Matrices for Passive Force Control

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Abstract

Robot force control implemented by means of passive mechanical devices has inherent advantages over active implementations with regard to stability, response rapidity, and physical robustness. The class of devices considered in this paper consists of a Stewart platform-type mechanism interconnected with a network of adjustable mechanical elements such as springs and dampers. The control law repertoire of such a device, imagined as a robot wrist, is given by the range of admittance matrices that it may be programmed to possess. This paper focuses on wrists incorporating damper networks for which the admittance matrices reduce to accommodation or inverse-damping matrices.

We show that a hydraulic network of fully adjustable damper elements may attain any diagonally dominant accommodation matrix. We describe the technique of selecting the individual damping coefficients to design a desired matrix. We identify the set of dominant matrices as a polyhedral convex cone in the space of matrix entries, and show that each dominant matrix can be composed of a positive linear combination of a fixed set of basis matrices.

The overall wrist-accommodation matrix is obtained by projecting the accommodation matrix of the damper network through the wrist kinematics. The linear combination of the dominant basis matrices projected through the wrist kinematics generates the entire space of mechanically implementable force-control laws. We quantify the versatility of mechanically implementable force-control laws by comparing this space to the space of all matrices.

KEY WORDS—automated assembly, passive programmable wrist, RCC, Stewart platform, hydraulic network, accommodation matrix

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1. Motivation and Background

In precision tasks such as robotic assembly, force control seems to be the natural choice and is widely believed to be superior to pure position control (Ang and Andeen 1995; Trong, Betemps, and Jutard 1995; Hogan 1985; Whitney 1987). In a typical force-control scheme, the motion of a robot is guided, according to a predefined control law, by the forces the robot encounters while interacting with the environment. The performance of such a scheme depends on the particular force-control law and the nature of its implementation.

Force-control laws may be broadly classified into two types: passive laws and active laws. A passive law describes a force-motion behavior that may, in principle, be exhibited by some passive physical system. Active laws, on the other hand, require the presence of a power source in the system. Since a passive system is guaranteed stable (Desoer and Kuh 1969), a passive control law, mimicking a passive system, is also stable. While an actively controlled system may certainly be stable, it is *only* a passive system that remains stable at all frequencies while interacting with arbitrary passive environments (Colgate and Hogan 1988) that are typical in robotic assembly.

A passive force-control law may be implemented either by a software algorithm or by an unpowered mechanical system. In a software-controlled system, active components (such as motors) are controlled in such a way that the overall system emulates a passive behavior (Anderson 1990; Chapel and Su 1992; Newman and Dohring 1991; Wang and Vidyasagar 1990). Unfortunately, the speed and performance of such a system is limited by the control-system bandwidth (Whitney 1987), force-feedback gain (Hogan and Colgate 1989), response time of the actuators, and noncollocation of the sensors and actuators (Eppinger 1988).

Unpowered devices with fixed mechanical properties lack the versatility offered by software controllers. An attractive alternative for implementing force-control laws is the use of passive mechanical devices with user-programmable properties. Such a device is able to regain some of the versatility of its active counterpart. Rather than involving the whole robot arm for the fine positionings necessary for the completion of most assembly tasks, using a low-inertia robotic wrist mounted at the end of the robot arm will have the advantage of higher mechanical bandwidth (Sharon, Hogan, and Hardt 1989). This was demonstrated by the success of the remote center of compliance (RCC) wrist in peg-in-hole assembly (Drake 1977; Whitney 1982). Biological evolution also seems to have taken notice of this fact, as is apparent in human manipulation. High-power tasks that do not a require high bandwidth or a high dexterity (e.g., pushing a heavy table, swinging a baseball bat) generally directly involve the powerful muscles of the upper arm. Low-power tasks requiring a high bandwidth (such as typing) and/or a high dexterity (such as writing) tend to decouple the heavier upper arm and, instead, use the low-inertia fingers (Cutkosky and Wright 1986).

Recently we have noted a renewed interest in passivity (Charles 1994; Davis and Book 1997) in such diverse areas as haptic displays (Peshkin, Colgate, and Moore 1996), medical robotics (Troccaz and Lavalle 1993), and exercise machines (Li and Horowitz 1995), in addition to applications in automated assembly (Gershon 1994; Ang and Andeen 1995). This work falls in the general category of research that seeks to quantitatively characterize passive devices in terms of their limitations and utilities. In this paper we focus on the use of the programmable passive mechanical robot wrist, which by virtue of its inherent mechanical properties, allows a simple and robust implementation of stable and fast force-control laws.

1.1. Conceptual Design of a Programmable Passive Wrist

The passive wrist considered in this paper consists of a set of unpowered hydraulic cylinders with their ports interconnected via a hydraulic network of programmable damping constrictions. A simple sketch of such a wrist possessing only 2 DOF is shown in Figure 1. The wrist consists of two hydraulic cylinders in a parallel configuration. The adjustable constrictions of the interconnecting hydraulic network allow one to "program" a desired accommodation (inverse-damping) matrix. Although robotic arms have some structural accommodation properties, the accommodation of the end-point device is assumed to be higher and to dominate the overall accommodation.

Figure 2 shows a more realistic example—a six-cylinder hydraulic wrist with a parallel manipulator geometry, often generically called a Stewart platform. The base of the wrist is attached to the main robot body and the platform extends

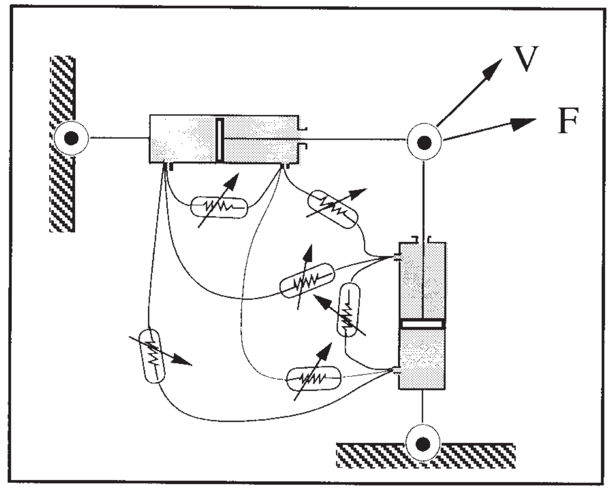

Fig. 1. A simple parallel 2-DOF passive mechanism. The ports of the hydraulic cylinders are interconnected through a network of tunable dampers. The accommodation of the mechanism, i.e., the physical relationship between $\boldsymbol{V}$ and $\boldsymbol{F}$, depends on the damper values.

to the gripper. The interconnection topology of the damper element network presented in Figure 1 is called the *fully connected lattice* pattern, and is considered to be the most general network for realizing accommodation matrices (Cederbaum 1958). This interconnection topology is repeated for every pair of cylinders for the wrist in Figure 2. The 6-DOF wrist therefore requires a total of 66 damper elements. Although robot arms have some inherent structural accommodation properties, the accommodation of the end-point device will be much higher, and will dominate the overall accommodation, especially for nonbackdrivable robots (Trong, Betemps, and Jutard 1995).

The accommodation matrix that a wrist imparts to a rigidly held workpart is called its *task-space accommodation matrix*. This matrix depends on the hydraulic conductance matrix of the network (the *joint-space accommodation matrix*), as well as the spatial layout of the cylinders. Two factors in turn contribute to the joint-space accommodation matrix: the values of the individual damper elements, and the topology of their interconnection.

Since we have a fixed choice of the damper-network topology, the only other means to program an accommodation matrix is either through the cylinder layout (i.e., the kinematics of the wrist), or by modifying the damper-element values. Also, since for a given assembly task the wrist executes small motions which do not significantly influence its spatial geometry during the task, this paper assumes, in addition, a fixed nominal configuration of the wrist. Thus to program an accommodation matrix we only explore the possibility of varying the damper elements. In contrast, in the approach taken

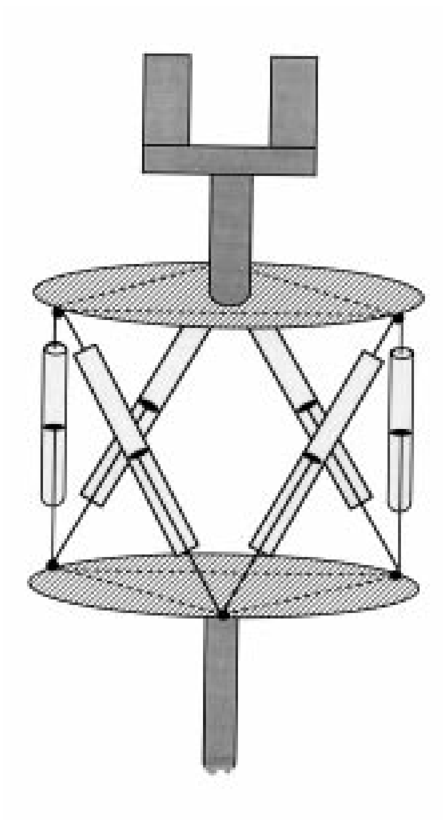

Fig. 2. A 6-DOF Stewart platform-type robot wrist. Every pair of cylinders is connected through a fully connected network of tunable dampers. The overall accommodation of the wrist may be "programmed" by carefully selecting the damper values.

by Charles (1994) and Davis and Book (1997), the values of the damper elements were held fixed, whereas the coupling between the degrees of freedom were controlled. Control of coupling may be obtained in our context, either by changing the network topology or by extreme cases of the damper values.

A brief description of the accommodation control law, the class of control laws which we intend to implement through passive hydraulic mechanical devices, follows next.

1.2. Accommodation Control Law

Imagine that a workpiece is held by a wrist such as shown in Figure 2, and is moving with a nominal velocity $\boldsymbol{v}_0$ in the absence of any assembly force; $\boldsymbol{v}_0$ is therefore the velocity of the robot/workpiece under pure position control. When the workpiece comes in contact with its mating part, its resultant velocity $\boldsymbol{v}$ may be expressed as

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{A}\boldsymbol{f}, \tag{1}$$

where $\boldsymbol{f}$ is the force resulting from unavoidable positional errors between the mating parts, and $\boldsymbol{A}$ is the accommodation matrix that maps forces imparted on the workpart to output velocities. Each of $\boldsymbol{v}$, $\boldsymbol{v}_0$, and $\boldsymbol{f}$ is a six-vector (translational and rotational velocities, or forces and torques), and $\boldsymbol{A}$ is a 6×6 matrix.

Equation (1) represents the force-control law that we intend to implement with programmable passive wrists. The control law is essentially an additive modification to the nominal velocity $\boldsymbol{v}_0$ of the wrist. The deviation of the wrist motion from the nominal velocity (given by $\boldsymbol{A}\boldsymbol{f}$) is a function of the task-space accommodation matrix $\boldsymbol{A}$ of the wrist. The success of the control strategy lies in the proper choice of $\boldsymbol{A}$ such that the resultant velocity $\boldsymbol{v}$ reduces relative positional errors between the mating parts. The passive wrist under discussion is programmed to possess the chosen $\boldsymbol{A}$.

The force-velocity model adopted in eq. (1) is also known as the *generalized damper model* of a system. This is to be contrasted with the force-displacement model called the generalized spring model that was adopted in the research behind the RCC wrist (Loncaric 1987). We have demonstrated the utility of programming a manipulator's linear damping characteristics so that for some classes of assembly tasks the forces that arise from positional errors of the mating parts in assembly naturally result in the motions that correct the errors. These types of tasks, which we call *force-guided assembly* (Peshkin, Goswami, and Schimmels 1993), can be performed under force control alone, with no other sensory information (Peshkin 1990; Schimmels and Peshkin 1990).

1.3. Problem Statement and Summary of Approach

As a demonstration of the utility of programmable passive devices, we showed (Goswami, Peshkin, and Colgate 1990) that an unpowered hydraulic wrist can be programmed to possess a *center of accommodation* (analogous to a *center of compliance*) anywhere in a substantial volume of space around it.

An accommodation matrix does not necessarily need to have a center to be useful in assembly operations, and often they do not (Schimmels and Peshkin 1992; Ang and Andeen 1995). In the current work, we characterize the complete range of task-space accommodation matrices, diagonalizable (i.e., with a center) and otherwise, that may be mechanically implemented by a programmable passive wrist.

Accommodation matrices which are *in principle* attainable with a network of passive dampers are called *realizable* matrices, according to network theory (Weinberg 1962). However, realizable accommodation matrices exist for which no routine way of computing the necessary network parameters is available. By *synthesizable* matrices, we refer to those matrices that the wrist can be systematically (algorithmically) programmed to possess.

In the present context, the set of matrices obtained by projecting the synthesizable matrices to the task space are the mechanically implementable accommodation matrices.

The approach presented in this paper can be summarized as follows:

1. In view of the physical analogy between the electrical and mechanical domains, we identify a network of tunable passive dampers as an electrical network of positive resistors. We adapt results from the electrical network theory, and determine that a certain class of matrices called the dominant matrices constitutes the class of synthesizable joint-space accommodation matrices of the passive wrist mechanism.
2. We project the dominant matrices through the wrist kinematics by means of congruence transformation to obtain the class of synthesizable task-space accommodation matrices. This represents the control-law repertoire that we may achieve with programmable passive dampers.
3. We show the technique of selecting the individual damping coefficients to achieve a desired control law.
4. The synthesizable matrices are shown to form a polyhedral convex cone in the space of matrix entries and each matrix can be composed of a positive linear combination of a fixed set of basis matrices.
5. To estimate the range of the passive control laws against the active laws, we compare the space of synthesizable matrices to a standard class of matrices (positive semidefinite matrices). This comparison tool can be graphically visualized for low-dimensional cases.
6. Finally, we detail the accommodation-matrix design procedure with step-by-step examples.

2. Synthesis of Joint-Space Matrices: Visualization and Comparison

Analogues exist among passive devices in different physical domains—electrical, mechanical, hydraulic, etc.—and may be exploited to model physical systems (Karnopp and Rosenberg 1975). Understanding these physical analogies makes it possible to apply results obtained in one physical domain to another.

2.1. Results from Electric Network Theory

A general representation of a linear dynamic system may be in terms of its admittance or impedance matrix. Dynamic behavior of *single-element-kind* mechanical systems may be expressed by special forms of admittance matrices. The behavior of a network of linear springs is expressed in terms of its compliance matrix. Similarly, the dynamic behaviors of a generalized damper and a generalized mass are described by accommodation matrices and inverse-inertia (or mobility) matrices, respectively.

Passive networks satisfy the so-called *passivity condition*, which implies that matrices adopted by passive devices must be positive real (Weinberg 1962). If we remove gyrators[1] from a general passive device, the admittance matrix must be symmetric (Anderson and Vongpanitlerd 1973; Desoer and Kuh 1969). If, in addition, we remove capacitors and inductors (or their mechanical analogues) from the possible range of available components, we are left with a positive semidefinite (PSD) matrix, which is a matrix with real entries.

The exclusion of transformers leaves us with a purely resistive circuit that possesses the so-called *no-amplification property*. Cederbaum (1958) generalized the idea of the no-amplification property, and showed that the accommodation matrix of a purely resistive circuit must be a *paramount matrix*.

DEFINITION 1. *Paramount matrix.* A real symmetric matrix is said to be *paramount* if any of its principal minors is not less than the absolute value of any other minor built on the same rows (or columns) by replacing any number of columns (or rows).

It has been shown that being paramount is a necessary but unfortunately not a sufficient condition for realizability.[2] There are presumably other restrictions on realizability that have not yet been identified. A sufficient, but overly restrictive, condition for an accommodation matrix to be attainable is that it be *dominant* (Weinberg 1962; Kim and Chen 1962).

DEFINITION 2. *Dominant matrix.* A real symmetric matrix is said to be *dominant* if each of its main diagonal entries is not less than the sum of the absolute values of all other entries in the same row (or column).

There are, in fact, examples of networks whose accommodation matrices are not dominant (Goswami 1993). There are also examples of paramount matrices for which it can be proved that there is no realization. Dominant matrices represent an important class of matrices in the synthesis of passive resistive networks, because there is a methodical procedure for synthesizing *any* dominant matrix. Therefore for our purpose, dominant matrices are classified as the synthesizable matrices.

2.1.1. Synthesis of a General Dominant Matrix (Weinberg 1962; Kim and Chen 1962)

It is known that a network of $2n$ nodes or junction points is the most general network for realizing an $n \times n$ accommodation matrix, where n is the number of hydraulic cylinders (Cederbaum 1958). Figure 3 shows the arrangement of dampers in the hydraulic network connecting the ith cylinder and the kth cylinder; F_i and F_k are the forces applied on the respec-

1. Gyrators are one of the five fundamental passive elements; the other four are resistors, capacitors, inductors, and transformers (Karnopp and Rosenberg 1975).
2. An exact necessary and sufficient condition exists for realizability, but testing a matrix for realizability using this condition is intractable (Civalleri 1968).

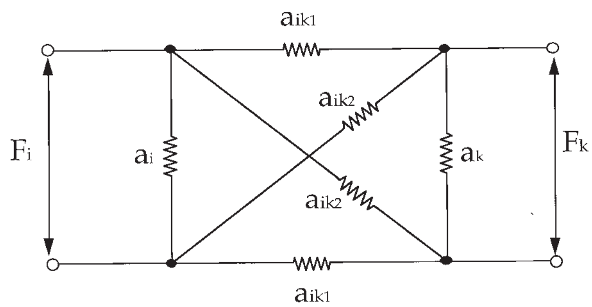

Fig. 3. The hydraulic network of dampers connecting the ith cylinder and the kth cylinder of a Stewart platform-type assembly wrist. This network pattern, repeated for every pair of cylinders, lets one synthesize any dominant matrix in the joint space.

tive cylinders. The coefficients of accommodation (inverse of damping coefficient) of the damper elements in the figure are given by

$$a_i = A_{ii} - \sum_{\substack{m=1\\ m\neq i}}^{n} |A_{im}|, \qquad a_k = A_{kk} - \sum_{\substack{m=1\\ m\neq k}}^{n} |A_{km}|, \tag{2}$$

$$a_{ik1} = |A_{ik}| - A_{ik}, \qquad a_{ik2} = |A_{ik}| + A_{ik}, \tag{3}$$

where A_{ik} is the entry of the ith row and kth column of the desired accommodation matrix $\boldsymbol{A}$.

The pattern of interconnection between the ith and the kth cylinders, sometimes called the *fully connected lattice pattern*, is repeated for every pair of cylinders. It can be shown that depending on the sign of the off-diagonal terms of the accommodation matrix, the circuit reduces to two parallel arms of equal accommodation, a_{ik1}, or the two cross-arms of equal conductance, a_{ik2}. A zero off-diagonal term implies a decoupling of the respective cylinders, and from eq. (3), by setting $A_{ik} = 0$, one observes that the coefficients of accommodation of both the parallel and the cross-arms become zero. This is equivalent to disconnecting those branches from the network. For realizing an $n \times n$ accommodation matrix, therefore we need $n(2n-1)$ dampers, although a maximum of n^2 of those are used to synthesize a particular accommodation matrix.

2.2. Dominant Basis Matrices

We have discovered a particularly useful property of dominant matrices which proves to be very useful in characterizing the range of mechanically implementable control laws. We first state the property, as follows.

PROPERTY 1. Any $n \times n$ dominant matrix can be expressed as a non-negative linear combination of a basis set of n^2 dominant matrices. These we call the *dominant basis matrices*.

The basis matrices can be compared with the basis vectors spanning a linear vector space where any arbitrary vector can be expressed as a linear combination (positive and negative) of the basis vectors. According to the above property, any $n \times n$ dominant accommodation matrix $\boldsymbol{A}$ may be expressed as

$$\boldsymbol{A} = \sum_{i=1}^{n^2} \alpha_i \boldsymbol{A}_i, \tag{4}$$

where the α_i are non-negative scalar coefficients, and the $\boldsymbol{A}_i$ are the dominant basis matrices.

As an example, let us take a general 3×3 dominant matrix $\boldsymbol{A}$ of the form $\begin{pmatrix} a & b & d \\ b & c & e \\ d & e & f \end{pmatrix}$.

Dominance requires the following:

$$a \geq |b| + |d|, \tag{5}$$

$$c \geq |b| + |e|, \tag{6}$$

$$[3pt] f \geq |d| + |e|. \tag{7}$$

Following eq. (4), matrix $\boldsymbol{A}$ may be expressed as a non-negative linear combination of the dominant basis matrices $\boldsymbol{A}_{1\ldots9}$,

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{A}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$\boldsymbol{A}_4 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{A}_5 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_6 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \tag{9}$$

$$\boldsymbol{A}_7 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

$$[3pt] \boldsymbol{A}_9 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \tag{10}$$

along with the non-negative coefficients

$$\alpha_1 = a - (|b| + |d|), \quad \alpha_2 = c - (|b| + |e|),$$
$$\alpha_3 = f - (|d| + |e|), \tag{11}$$

$$\alpha_4 = \frac{|b| + b}{2}, \qquad \alpha_5 = \frac{|b| - b}{2},$$
$$\alpha_6 = \frac{|d| + d}{2}, \tag{12}$$

$$\alpha_7 = \frac{|d| - d}{2}, \qquad \alpha_8 = \frac{|e| + e}{2},$$
$$\alpha_9 = \frac{|e| - e}{2}. \tag{13}$$

We note that each dominant basis matrix is PSD of rank 1; i.e., each has only one positive eigenvalue. Also we observe that although n^2 dominant basis matrices are necessary to represent the full range of $n \times n$ dominant matrices, for any given $n \times n$ matrix, we need only a set of $\frac{n(n+1)}{2}$ basis matrices. This is because, depending on the sign of the off-diagonal entries, some of the α_i become zero.

2.3. Space of Dominant Matrices

The entire space of dominant matrices is indicative of the range of control laws implementable by the passive network of dampers. The characterization of the volume of dominant matrices in a matrix space, which we do in this section, is facilitated by the use of dominant basis matrices. First we explain the idea behind the characterization of matrix spaces as adopted in this paper.

Let us associate each $n \times n$ dominant matrix with a point in $\mathcal{R}^{n(n+1)/2}$, where $n(n+1)/2$ is the number of distinct entries in an $n \times n$ symmetric matrix. The definition of dominance translates to a set of inequality constraints that must be satisfied by the entries of the matrix. The portion of $\mathcal{R}^{n(n+1)/2}$ delimited by these constraints represents the space of dominant matrices.

Since any non-negative multiple of a dominant matrix is a dominant matrix itself, it is clear that the space of dominant matrices must be a cone. Indeed, the space of all $n \times n$ dominant matrices represents a polyhedral convex cone (PCC) in $\mathcal{R}^{n(n+1)/2}$. The cone has n^2 edges, each corresponding to one of the dominant basis matrices. The edges of the PCC coincide with the boundary of the cone representing PSD matrices. The representation of certain classes of matrices as cones is well known in linear algebra (Hill and Waters 1987).

For example, for 2×2 dominant matrices, we have the following four dominant basis matrices: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Each of these basis matrices corresponds to a point along one of the rays defining the edge of the PCC, namely, the points (1, 1, 1), (1, -1, 1), (1, 0, 0), and (0, 0, 1). See Figure 4 for a sketch of the PCC representing the characteristic volume of 2×2 dominant matrices and the four rays generating the dominant PCC. For higher-order matrices, our analysis remains valid, although the graphic visualization becomes impossible.

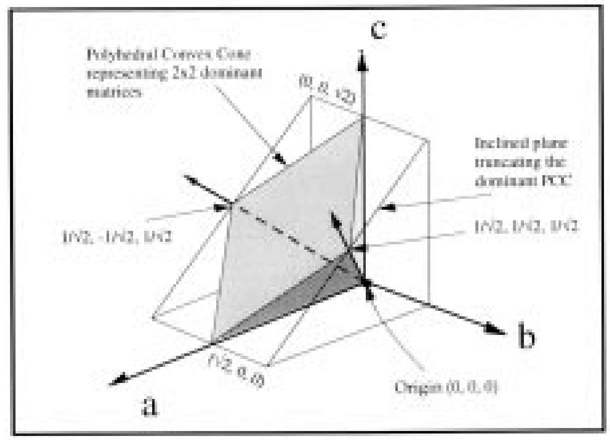

Fig. 4. The polyhedral convex cone (PCC) representing the entire range of 2×2 dominant matrices is shown. The PCC is truncated by a 45° inclined plane.

2.4. Comparison between the Spaces of Dominant and PSD Matrices

To obtain a measure of the space of dominant matrices, we compare it with the space of PSD matrices. PSD matrices represent the largest class of matrices that we might hope to synthesize with a gyratorless passive system. This implies, as we verify later, that the set of dominant matrices forms a proper subset of PSD matrices. In addition, PSD matrices are well studied, and a feel for their character and range already exists. By comparing dominant matrices with them would place the class of dominant matrices in a known perspective.

The set of $n \times n$ PSD matrices is known to represent an infinite cone in $\mathcal{R}^{n(n+1)/2}$ (Hill and Waters 1987). To compare its volume with that of the dominant PCC, we first analyze a low-dimensional example with the help of graphical representation, and generalize the results to the higher dimension.

A sufficient condition for a symmetric matrix to be PSD is that the determinant of each of its principal submatrices is non-negative. Applying this condition to a general symmetric matrix leads to a set of inequalities that must be satisfied by the matrix entries. For the 2×2 PSD matrix $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$, the inequalities are

$$a \geq 0 \qquad \text{and} \qquad ac - b^2 \geq 0. \tag{14}$$

The complete set of (a, b, c) that satisfies the above conditions lies within a cone with an elliptical cross-section touching the a-axis and the c-axis; see Figure 5. The vertex of

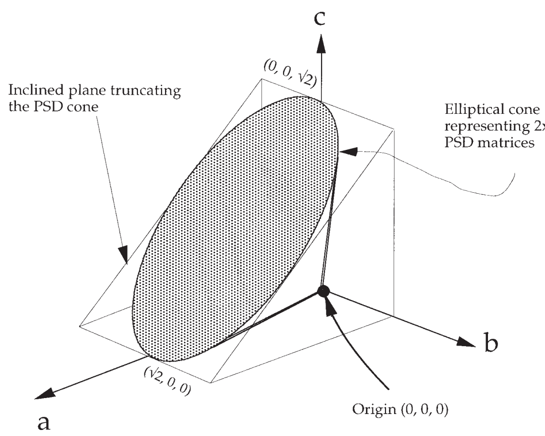

Fig. 5. The elliptical cone representing the entire class of 2×2 PSD matrices is shown. The cone is truncated by a 45° inclined plane to reveal its elliptical cross-section.

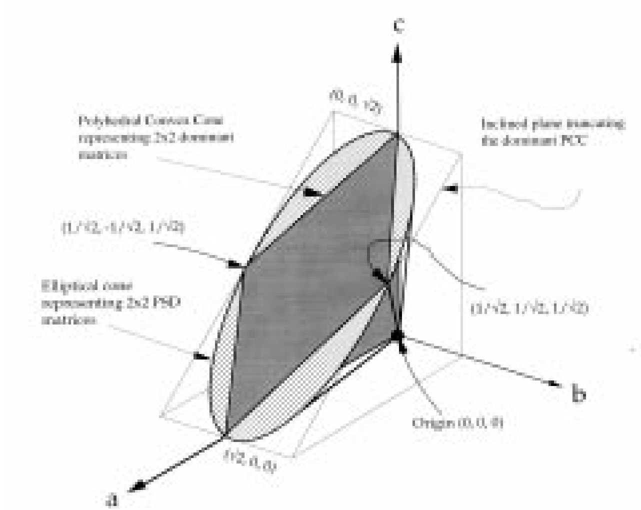

Fig. 6. The superposition of Figures 4 and 5. This shows that the dominant PCC is completely inside the PSD cone, implying that dominant matrices are a proper subset of the PSD matrices.

this cone is at the origin $(0, 0, 0)$. Any point inside the cone represents a positive-definite matrix (which is also positive *semi*definite by definition), whereas the boundary points of the cone represent the *strictly* PSD matrices, one of whose eigenvalues must be zero. Generalizing, the space of all $n \times n$ symmetric PSD matrices is a hypercone in $\mathcal{R}^{n(n+1)/2}$.

Since each dominant basis matrix as well as its non-negative multiples are strictly PSD, the edges of the dominant PCC must coincide with the boundary of the PSD cone. The entire dominant PCC must therefore lie within the PSD cone as we confirm by superposing the cones; see Figure 6. We compare the overall sizes of the cones by comparing their footprints on the same intersecting plane. This is reasonable, since volumes of cones of the same height are proportional to their footprints. In Figure 6, the intersecting plane is at unit-normal distance from the origin.

The set of all 6×6 PSD matrices therefore represents a cone in $\mathcal{R}^{21}$, and its intersection with a 20-dimensional hyperplane perpendicular to its axis gives rise to a hyperellipsoid in $\mathcal{R}^{20}$. The set of all 6×6 dominant matrices, on the other hand, represents a PCC in a $\mathcal{R}^{21}$. This PCC has 36 edges, and its intersection with a 20-dimensional hyperplane generates a polytope in $\mathcal{R}^{20}$ with 36 vertices. Since each dominant basis matrix is strictly PSD, the vertices of this polytope lie on the boundary of the PSD hyperellipsoid.

3. Synthesizable Task-Space Matrices

The procedure for realizing an accommodation or a compliance matrix for a particular task is most naturally undertaken in the joint space, as we saw in the last section. However, the most convenient way of describing a matrix suitable for a given task is in terms of the task-space variables (Schimmels and Peshkin 1992; Ang and Andeen 1995). The task-space and joint-space accommodation matrices, $\boldsymbol{A}_t$ and $\boldsymbol{A}_j$, respectively, are related by

$$\boldsymbol{A}_t = ({}^t_j\boldsymbol{J})\boldsymbol{A}_j({}^t_j\boldsymbol{J}^T), \tag{15}$$

where ${}^t_j\boldsymbol{J}$ is the wrist Jacobian.

The above equation is an example of *congruence transformation* between $\boldsymbol{A}_j$ and $\boldsymbol{A}_t$. A congruence transformation, according to *Sylvester's law of inertia* (Horn and Johnson 1985), preserves the "inertia" of a matrix. Inertia in this context is an ordered triple representing the numbers of positive, negative, and zero eigenvalues of a matrix. This law of inertia implies that the cone representing the PSD matrices is invariant under congruence transformation; i.e., no matter what Jacobian is used, the joint-space PSD cone and the task-space PSD cone are identical. The interior and boundary points of the joint-space cone map respectively to interior and boundary points of the task-space cone. Intuitively, since positive semidefiniteness is associated with the basic requirements of passivity of a network, a device that is passive in the joint space is expected to remain passive in the task space as well.

The property of dominance is not preserved under the above congruence transformation. From a physical standpoint, this can be understood by the fact that the manipulator links are mechanical equivalents of electrical transformers. Therefore, when the hydraulic network is viewed from the task space, it is a network of resistances *and* transformers, having the capability of possessing any PSD matrix, given the full flexibility of transformer parameters (which for a manipulator are functions of the link lengths and the joint angles).

To take full advantage of this, the design of the assembly wrist should be carefully planned.

How does the *entire* range of joint-space dominant matrices transform to the task space? The answer may be logically presented as in the following. Since each dominant basis matrix is strictly PSD in the joint space, they remain so after they are projected to the task space. These projected matrices constitute the edges of the synthesizable task-space PCC. The boundary of the task-space PSD cone therefore contains the edges of the synthesizable task-space PCC. Consequently, the vertices of the polytope obtained by truncating the synthesizable task-space PCC with a hyperplane are on the boundary of the intersection of the task-space PSD cone with the same hyperplane. However, the synthesizable PCC in the joint space and in the task space do not, in general, have the same shape; this depends on the manipulator Jacobian.

4. Examples

In the rest of the paper, the range of mechanically implementable accommodation matrices are computed for several manipulators. We first identify the set of passive accommodation control laws for simple 2-DOF manipulators for which the results may be graphically visualized. Next we take up the example of a planar 3-DOF assembly wrist, for which some results may be graphically presented. Finally, we provide guidelines for accommodation matrix design for full 6-DOF wrists.

4.1. Range of Control Laws for 2-DOF Mechanisms

Let us consider a 2-DOF passive parallel mechanism, the cylinders of which are interconnected by the fully connected network discussed in Section 2.1, Figure 3. In Figure 7, we show the PSD ellipse and the joint-space dominant quadrilateral (in dashed lines) for two different configurations of such a mechanism. The joint-space quadrilateral is mapped through the mechanism's Jacobian, according to eq. (15), to obtain the task-space quadrilaterals shown (in solid lines) superposed in the figures.

We might want to determine the posture of the manipulator that gives us the maximum ranges of synthesizable matrices in the task space. For this example, the area of the PSD ellipse is $\frac{\pi}{\sqrt{2}}$, whereas the joint-space quadrilateral with an area of $\sqrt{2}$ is, curiously, the largest quadrilateral that may be inscribed into the ellipse. The joint-space and task-space quadrilaterals are identical when the Jacobian is an identity matrix. This happens when the joint angles are 0° and 90°.

Depending on the configuration of the mechanism, the dominant PCC for this example and in general may degenerately map into a much smaller task-space PCC. In the current example, the joint-space quadrilateral may reduce to a triangle, a line, or even a point in the task space. The reduction in the task-space PCC as the manipulator approaches a singularity is shown in Figure 9b.

Although this paper discusses robot wrists of parallel kinematics, passive devices of serial kinematics may also be useful as macro-manipulators (Charles 1994) or micromanipulators for assembly (Schimmels and Huang 1995). Without going into the details of how to physically interconnect the joints of a serial manipulator with a damper network, we may simply consider the effect of its Jacobian on the joint-space dominant matrices. We show this for two different configurations of a 2-DOF serial manipulator of unity link lengths (Fig. 8). In Figure 9a, we show the reduction in the area of the task-space quadrilateral as the serial manipulator approaches a singularity.

4.2. Center of Accommodation for Planar Wrists

Recall that the RCC wrist possesses a center of compliance near the tip of the rigidly held peg which is to be inserted into a chamfered hole. Over what range of space can we move the center of accommodation of our damper-based wrist, simply by selecting the damper-element values? To determine this, we need to choose a diagonal accommodation matrix at a point in the task space, transform it to the joint space, and test for the dominance of the resulting matrix. We assume that the fixed-topology fully connected network interconnects each pair of cylinders, and that the configuration of the wrist is kept fixed during an assembly task.

By analogy to the term "forward kinematics," the computation of the task-space accommodation matrix from the given joint-space matrix may be called the *forward-accommodation transformation* problem. This is relevant when we need to characterize the range of mechanically implementable accommodation matrices, as is done in this paper. The reverse problem, that of determining the joint-space accommodation matrix from a desired task-space matrix, may be called the *inverse-accommodation transformation* problem. The inverse-accommodation transformation problem is relevant when a desired control law, expressed by means of a task-space accommodation matrix, needs to be implemented. We have already seen the forward-accommodation transformation relationship in eq. (15). The inverse-accommodation transformation relationship is given by inverting that equation as

$$\boldsymbol{A}_j = ({}^j_t\boldsymbol{J})\boldsymbol{A}_t({}^j_t\boldsymbol{J}^T), \tag{16}$$

where ${}^j_t\boldsymbol{J}^T = ({}^t_j\boldsymbol{J}^T)^{-1}$, which always exists for nonredundant manipulators in nonsingular configurations. The above procedure is conducted for a mesh of points around the planar wrist, considering different diagonal task-space accommodation matrices at each selected point. Although it is difficult to present in a compact form an exhaustive analysis for this mechanism, one representative example may easily demonstrate the basic procedure.

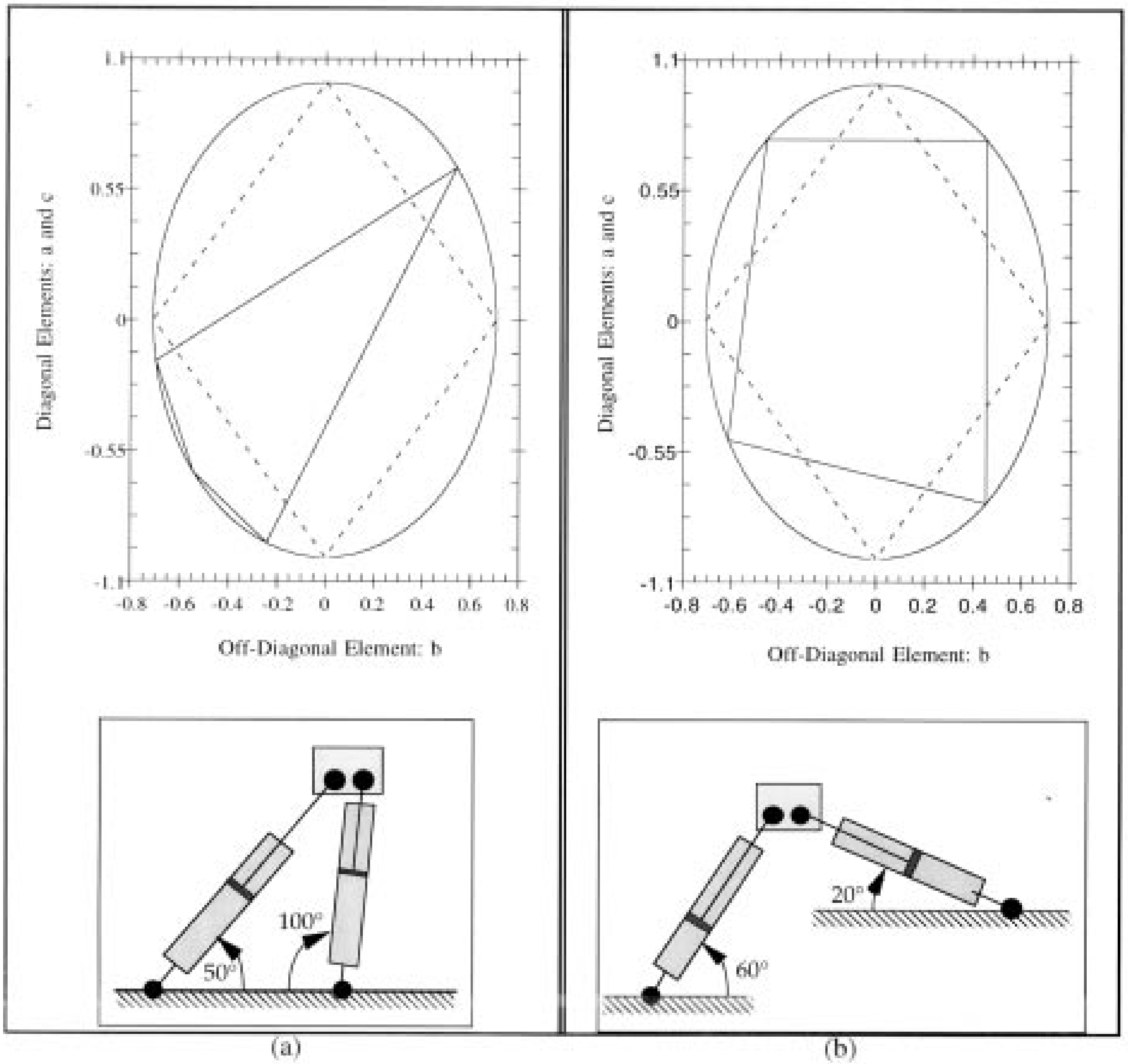

Fig. 7. Two different configurations (bottom) and the corresponding synthesizable accommodation matrices (top) of a 2-DOF planar parallel manipulator. The ranges of synthesizable matrices are represented by the quadrilaterals inscribed in the PSD ellipse. The dominant quadrilateral (dashed line) is also shown for comparison.

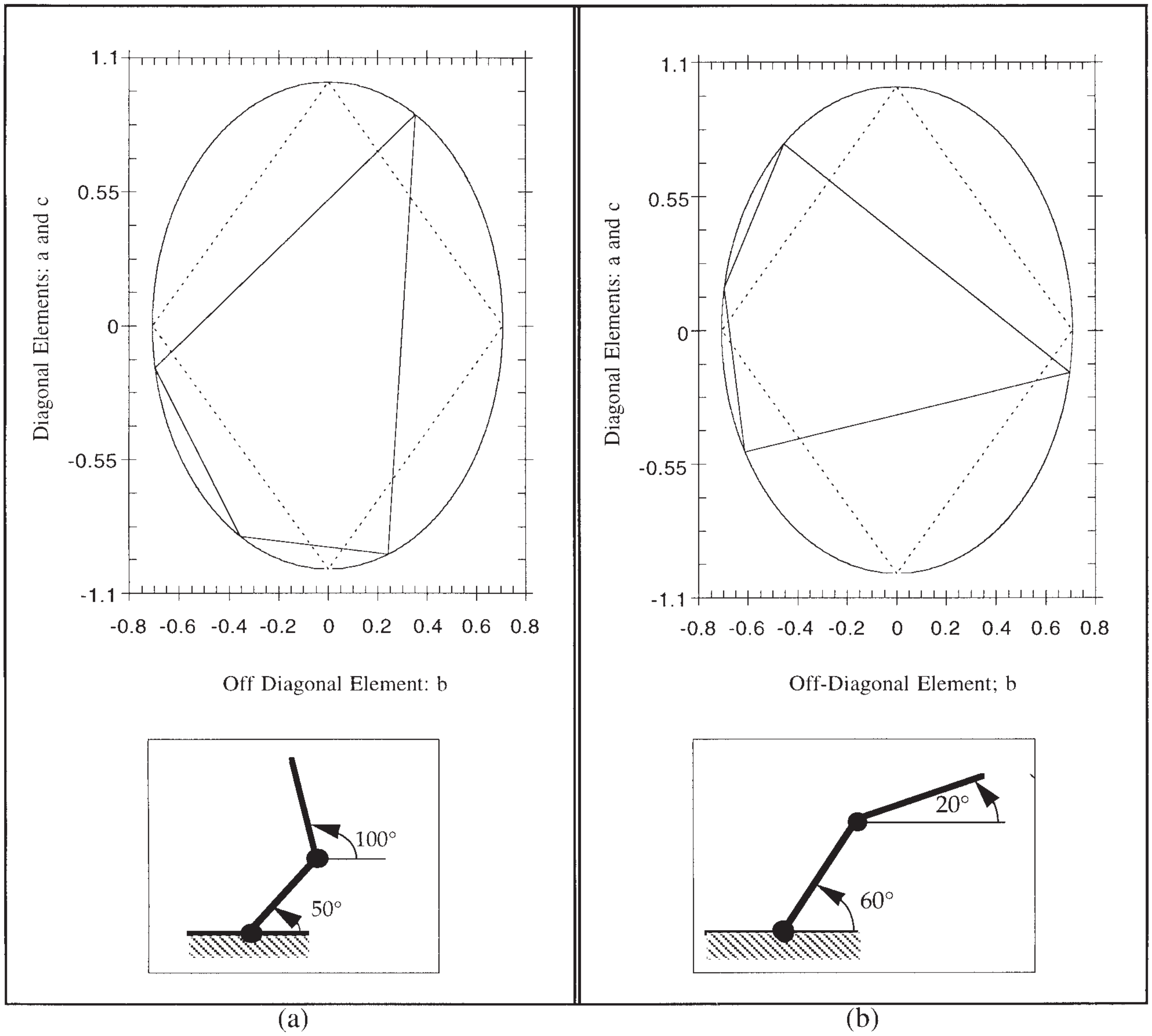

Fig. 8. An example analogous to that of Figure 7, for a 2-DOF serial manipulator.

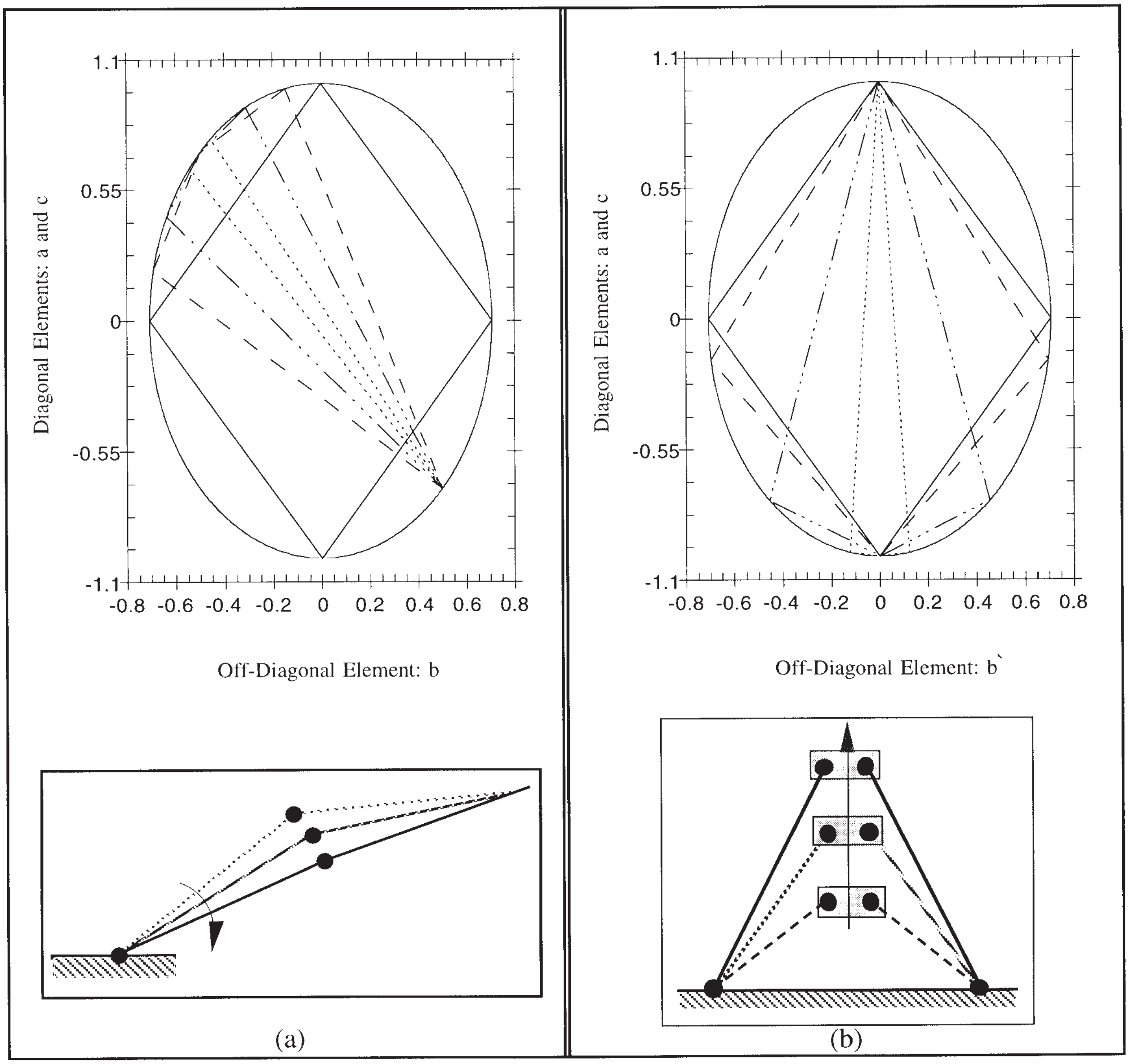

Fig. 9. When a manipulator approaches a singularity, the range of synthesizable matrices reduces. This is depicted for 2-DOF serial and parallel manipulators. In each figure, the three task-space quadrilaterals in dashed, dash-dot, and dotted lines represent the progressively diminishing range of synthesizable matrices.

Figure 10 presents a Stewart platform-type planar wrist comprising three cylinders. Both the top (the platform) and the bottom (the base) plates are of unit lengths—the length of a plate being the distance between the cylinder attachment points. In the nominal configuration, each cylinder is of unit length, and they make 60° angles with each other at the attachment points. At this configuration, the wrist may project its center of accommodation at any point in the region indicated as shaded in the figure.

The substantial expanse of the shaded region is demonstrative of the utility of programmable passive wrists. This is to be compared with the performance of the wrist reported by Cutkosky and Wright (1986), which may project its center of compliance on a line within a restricted region. It is not our objective to explore in detail the center of accommodation properties of the passive wrist, but to include matrices without "centers" as well. We do, however, mention one interesting fact: the shaded region in Figure 6 strongly depends on the nominal geometry of the wrist. In particular, we have verified with a few other configurations that this region appears to be completely determined by the cylinder orientations. Even for a 6-DOF wrist of spatial kinematics, this property remains valid. For the nominal configuration of such a wrist (shown in Fig. 2), the volume of the programmable center of accommodation is determined by the planes generated by the axes of the adjacent cylinders. Similar to the parallelogram-shaped area of the planar wrist, the spatial wrist exhibits a volume in the shape of a rhombic parallelepiped. Given the nature of the nonlinear inequalities involved, it is difficult to analytically describe the region of synthesizable accommodation centers. As an alternative, we adopted a computational search scheme.

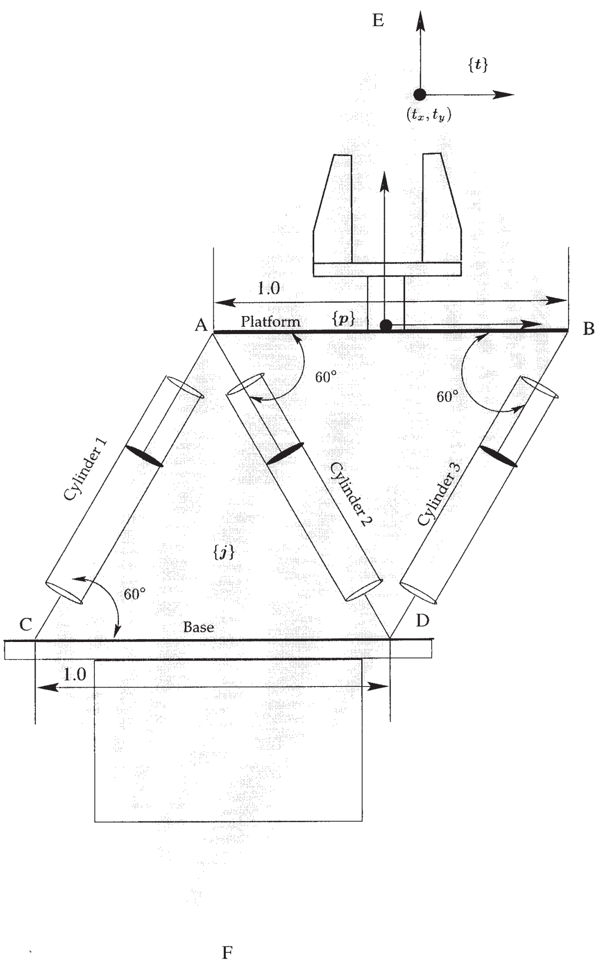

Fig. 10. The planar wrist may be programmed to place its center of accommodation at any point in the shaded region. In the depicted configuration, the platform, the base, and the three cylinders are all of unit length. The cylinders are oriented at 60° with each other as indicated. The shaded parallelogram is constructed out of four straight lines: EC, CF, FB, and BE. Lines EC and FB are coincident with the axes of the left and right cylinders, respectively; lines EB and CF are parallel to the axis of the middle cylinder (AD).

4.3. Accommodation-Matrix Design Technique

To carry out the detailed computation, three coordinate frames are set up for the transformation of the matrices: (1) the task-space frame $\{\boldsymbol{t}\}$, with axes X_t, Y_t; (2) the platform frame $\{\boldsymbol{p}\}$, with axes X_p, Y_p; and (3) the joint-space frame $\{\boldsymbol{j}\}$. The origin of $\{\boldsymbol{t}\}$ is located at (t_x, t_y), with respect to $\{\boldsymbol{p}\}$. The placement of $\{\boldsymbol{p}\}$ does not depend on the current task, and is generally kept fixed at a convenient position, making the transformation between $\{\boldsymbol{j}\}$ and $\{\boldsymbol{p}\}$ a constant. The placement of $\{\boldsymbol{t}\}$, however, depends on the current task. Following eq. (16), we may write

$$\boldsymbol{A}_j = \left(({}^j_p\boldsymbol{J})({}^p_t\boldsymbol{J}) \right) \boldsymbol{A}_t \left(({}^j_p\boldsymbol{J})({}^p_t\boldsymbol{J}) \right)^T, \tag{17}$$

where ${}^j_p\boldsymbol{J}$ and ${}^p_t\boldsymbol{J}$ are the Jacobian matrices from $\{\boldsymbol{j}\}$ to $\{\boldsymbol{p}\}$ and $\{\boldsymbol{p}\}$ to $\{\boldsymbol{t}\}$, respectively.

For the nominal configuration of the planar wrist, we have

$$\begin{aligned} {}^j_p\boldsymbol{J} &= \begin{bmatrix} 0.5 & 0.867 & -0.433 \\ -0.5 & 0.867 & -0.433 \\ 0.5 & 0.867 & 0.433 \end{bmatrix}, \\ {}^p_t\boldsymbol{J} &= \begin{bmatrix} 1 & 0 & t_y \\ 0 & 1 & -t_x \\ 0 & 0 & 1 \end{bmatrix}, \end{aligned} \tag{18}$$

and the shaded region corresponds to any $\boldsymbol{A}_t$ of the form

$$\boldsymbol{A}_t = \begin{bmatrix} a_{tx} & 0 & 0 \\ 0 & a_{ty} & 0 \\ 0 & 0 & a_{t\theta} \end{bmatrix} = \beta \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma \end{bmatrix}, \tag{19}$$

where $\beta \geq 0$ and $0 \leq \gamma \leq 8$.

As a concrete example, we consider a diagonal $\boldsymbol{A}_t$ with $a_{tx} = 3$, $a_{ty} = 1$, and $a_{t\theta} = 6$, which is considered at a task-space point, $t_x = 0.1$, $t_y = 0.6$. The corresponding $\boldsymbol{A}_j$ according to eq. (17) is

$$\boldsymbol{A}_j = \begin{bmatrix} 1.79 & 1.08 & 0.65 \\ 1.08 & 5.53 & -3.18 \\ 0.65 & -3.18 & 4.01 \end{bmatrix}. \qquad (20)$$

In order for the joint space of the wrist to possess this $\boldsymbol{A}_j$, we need three fully connected networks, each interconnecting a pair of cylinders. The accommodation values of the damper elements are computed using eqs. (2) and (3). We illustrate the network interconnection in Figure 11. The values of accommodation (m/N-sec, in the SI system) of the damper elements are shown on the figure.

Significant insight into eq. (17) may be obtained by noting that the ith row of ${}^j_p\boldsymbol{J}$ is $[\; l_{ix} \quad l_{iy} \quad \boldsymbol{r}_i \times \boldsymbol{l}_i \;]$, where $\boldsymbol{l}_i = l_{ix}\hat{i} + l_{iy}\hat{j}$ is the direction vector of the ith cylinder axis, and $\boldsymbol{r}_i$ is the vector from the origin of $\{\boldsymbol{p}\}$ to the attachment point of the ith cylinder with the top plate. For the specified nominal configuration, $l_{1x} = l_{3x} = \cos(60°)$, $l_{2x} = \cos(120°)$, $l_{1y} = l_{2y} = l_{3y} = \sin(60°)$, $\boldsymbol{r}_1 \times \boldsymbol{l}_1 = \boldsymbol{r}_2 \times \boldsymbol{l}_2 = -0.5\sin(60°)$, and $\boldsymbol{r}_3 \times \boldsymbol{l}_3 = 0.5\sin(60°)$.

By denoting $s = \cos(60°)$, $q = \sin(60°)$, and $w = 0.5\sin(60°)$, we may show that the joint-space accommodation matrix $\boldsymbol{A}_j$ may be written in the form

$$\boldsymbol{A}_j = a_{tx}\boldsymbol{M}_x + a_{ty}\boldsymbol{M}_y + a_{t\theta}\boldsymbol{M}_\theta, \qquad (21)$$

where

$$\boldsymbol{M}_x = \begin{bmatrix} s^2 & -s^2 & s^2 \\ -s^2 & s^2 & -s^2 \\ s^2 & -s^2 & s^2 \end{bmatrix},$$

$$\boldsymbol{M}_y = \begin{bmatrix} q^2 & q^2 & q^2 \\ q^2 & q^2 & q^2 \\ q^2 & q^2 & q^2 \end{bmatrix}, \qquad (22)$$

$$\boldsymbol{M}_\theta = \begin{bmatrix} k & k & k \\ m & m & m \\ n & n & n \end{bmatrix} \begin{bmatrix} k & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & n \end{bmatrix},$$

where

$$k = (t_x q - t_y s + w), \qquad m = (t_x q + t_y s + w),$$
$$n = (t_x q - t_y s - w). \qquad (23)$$

4.4. Guidelines for General Spatial Mechanisms

The procedure detailed in Section 4.3 may be used for a general class of matrices (diagonalizable and those that cannot be diagonalized) as well as for wrists with more degrees of freedom, although the involved equations will be more complicated.

The two basic questions regarding the synthesis of accommodation matrices are related to the forward- and inverse-accommodation transformations of matrices. The first

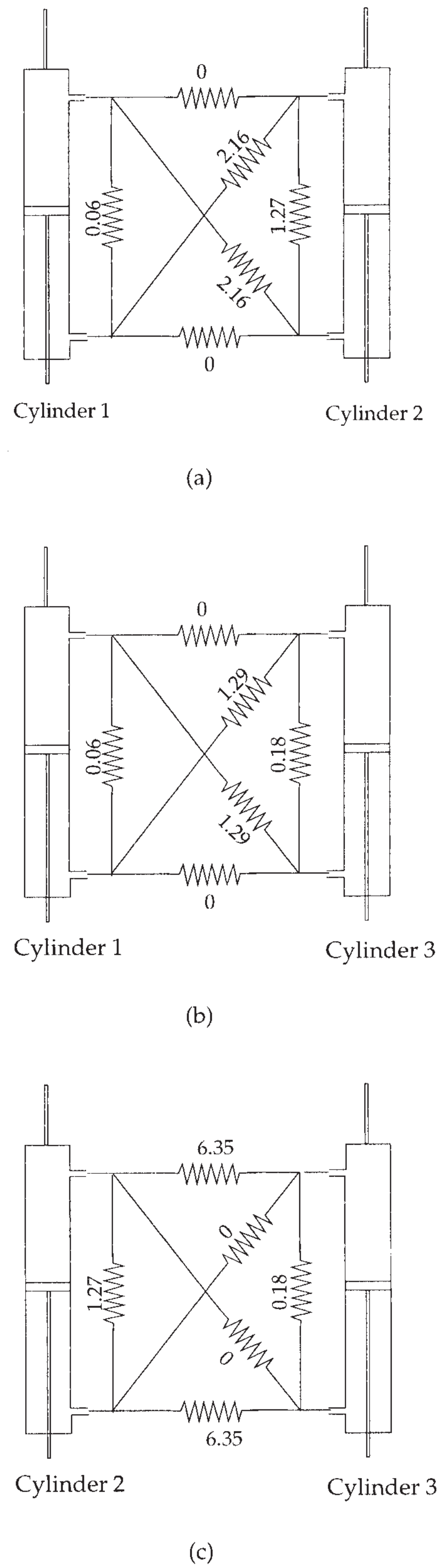

Fig. 11. The interconnecting network and the damper element values needed to attain the $\boldsymbol{A}_j$ given in eq. (20).

question, readily answered by the inverse-accommodation transformation equation seeks to compute the joint-space accommodation matrix for a desired task-space matrix. If the joint-space matrix is dominant, it is synthesizable; otherwise, it is not. It may be possible to attain a nondominant matrix, but a systematic synthesis procedure does not exist.

The second question addresses the opposite issue, involving the forward-accommodation transformation equation. The objective is to characterize the range of task-space accommodation matrices obtained from dominant joint-space matrices. For this, each dominant-basis matrix is mapped to the task space through eq. (15). Non-negative linear combinations of these task-space matrices give rise to a PCC. At each task-space point, there exists an associated PCC representing the synthesizable matrices. An accommodation matrix, to be mechanically implementable, must reside within this PCC.

5. Summary and Open Issues

The thesis of this paper is that a passive robotic wrist, of fixed mechanical design, can be programmed to execute a wide range of force-control laws useful in automated assembly. In this paper, we conducted a systematic study to characterize the range of control laws (given by accommodation matrices) implementable by a passive hydraulic network of user-programmable damper constrictions. We used electrical network theory results to identify the accommodation matrices that are attainable in the joint space of the wrist. We then projected these matrices to the task space and compared the range of task-space matrices to the class of PSD matrices in an attempt to quantify the usefulness of passive devices.

Practical implementations of accommodation-control laws by means of passive dampers must consider a finite range of damper-element values. The synthesizable matrices corresponding to range-limited dampers occupy a subpart of the dominant PCC.

Serious consideration should also be given to the sensitivity of the task-space accommodation matrices with respect to the errors in the individual damper-element values. Sensitivity analysis may also be performed with respect to the parameters in the manipulator kinematics, such as the cylinder lengths and the joint angles.

An interesting theoretical question is the relationship between the region of the implementable center of accommodation and the wrist kinematics. Although an in-depth study of this phenomenon was beyond the scope of this work, the results indicated that it was possible to express this relationship in simple terms.

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